DOI: 10.13718/j.cnki.xdzk.2016.04.008

一类退化 Kirchhoff 方程基态解的存在性[①]

刘婷婷， 商彦英

， 400715

摘要：研究了在 $\mathbb{R}^3$ 上的一类退化的 Kirchhoff 方程

$$\begin{cases} -\left(b\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\Delta u + V(x)u = |u|^{p-2}u & x \in \mathbb{R}^3 \\ u \in H^1(\mathbb{R}^3),\ u > 0 & x \in \mathbb{R}^3 \end{cases}$$

利用变分法及分析方法，得到了它的一个正的基态解.

关 键 词：Kirchhoff 方程；基态解；变分法；Pohozaev 等式

中图分类号：O176.3 **文献标志码：A** **文章编号：**1673-9868(2016)04-0054-07

Kirchhoff

$$\begin{cases} -\left(b\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\Delta u + V(x)u = |u|^{p-2}u & x \in \mathbb{R}^3 \\ u \in H^1(\mathbb{R}^3),\ u > 0 & x \in \mathbb{R}^3 \end{cases} \tag{1}$$

$b > 0$，$2 < p < 6$. V ：

(V_1) $V(x) \in C^1(\mathbb{R}^3, \mathbb{R})$ $(\nabla V(x), x) \in L^{\infty}(\mathbb{R}^3) \cup L^{\frac{3}{2}}(\mathbb{R}^3)$

$$\frac{p-2}{2}V(x) - (\nabla V(x), x) \geqslant 0 \qquad \text{a. e. } x \in \mathbb{R}^3$$

$(\cdot, \cdot)$ $\mathbb{R}^3$ ；

(V_2) $x \in \mathbb{R}^3$，$0 < \inf\limits_{x \in \mathbb{R}^3} V(x) \leqslant V(x) \leqslant \lim\limits_{|x| \to \infty} V(x) = V_{\infty} < +\infty$；

(V_3) $\bar{c} > 0$，

$$\bar{c} = \inf_{u \in H^1(\mathbb{R}^3)\setminus\{0\}} \frac{\int_{\mathbb{R}^3} (|\nabla u|^2 + V(x)u^2)\mathrm{d}x}{\int_{\mathbb{R}^3} u^2 \mathrm{d}x}$$

， Kirchhoff ， $p \geqslant 4$. ，
$p > 2$. (1)

$$\begin{cases} -\left(a + b\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\Delta u + V(x)u = |u|^{p-2}u & x \in \mathbb{R}^3 \\ u \in H^1(\mathbb{R}^3),\ u > 0 & x \in \mathbb{R}^3 \end{cases} \tag{2}$$

$a, b > 0$. (2) [1-2]. [2] $3 < p < 6$ ， [1] $2 <$

① ：2015-09-17
： (11071267)； (XD5K2016C119).
： (1991-)， ， ， ， .
： ， .

$p<6$, [3] Kirchhoff . [1-3] ,
Kirchhoff , (1). :

定理 1 $b>0$, $2<p<6$ V $(V_1)-(V_3)$, (1)
.

(1) , :

$$I(u)=\frac{b}{4}\left(\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\right)^2+\frac{1}{2}\int_{\mathbb{R}^3}V(x)u^2\mathrm{d}x-\frac{1}{p}\int_{\mathbb{R}^3}(u^+)^p\mathrm{d}x$$

$u^+=\max\{u,0\}$.

$$\langle I'(u),v\rangle=b\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\int_{\mathbb{R}^3}(\nabla u,\nabla v)\mathrm{d}x+\int_{\mathbb{R}^3}V(x)uv\mathrm{d}x-\int_{\mathbb{R}^3}(u^+)^{p-1}v\mathrm{d}x\qquad\forall v\in E$$

$$E=\left\{u\in H^1(\mathbb{R}^3):\int_{\mathbb{R}^3}V(x)u^2\mathrm{d}x<+\infty\right\}$$

$$\|u\|=\left(\int_{\mathbb{R}^3}(|\nabla u|^2+V(x)u^2)\mathrm{d}x\right)^{\frac{1}{2}}$$

$u\in H^1(\mathbb{R}^3)$, $\|u\|_*=\left(\int_{\mathbb{R}^3}(|\nabla u|^2+u^2)\mathrm{d}x\right)^{\frac{1}{2}}$. , $\|\cdot\|_*$ $\|\cdot\|$.
, Kirchhoff

$$\begin{cases}-b\left(\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\right)\Delta u+V_\infty u=\lambda|u|^{p-2}u & x\in\mathbb{R}^3\\ u\in H^1(\mathbb{R}^3),\ u>0 & x\in\mathbb{R}^3\end{cases}\tag{3}$$

b,V_∞,λ , $2<p<6$. (3) I_λ^∞, $I_\lambda^{\infty'}(u)$. ,
:

定理 2 b,V_∞,λ , $2<p<6$, (3) .
, Nehari Pohozaev ,
:

$$M_\lambda^\infty=\{u\in E\backslash\{0\}:J_\lambda^\infty(u)=0\}$$

$$\begin{aligned}J_\lambda^\infty(u)=&\langle I_\lambda^{\infty'}(u),u\rangle+2\langle I_\lambda^{\infty'}(u),(x,\nabla u)\rangle=\\&2b\left(\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\right)^2+4\int_{\mathbb{R}^3}V_\infty u^2\mathrm{d}x-\frac{\lambda(p+6)}{p}\int_{\mathbb{R}^3}|u|^p\mathrm{d}x\end{aligned}$$

$m_\lambda^\infty=\inf\limits_{u\in M_\lambda^\infty}I_\lambda^\infty(u)$. m_λ^∞ :

引理 1 b,λ , $2<p<6$, :

(i) $\beta>0$, $\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\geqslant\beta$ $u\in M_\lambda^\infty$;

(ii) $m_\lambda^\infty>0$.

证 (i) $u\in M_\lambda^\infty$, Young Sobolev , .

(ii) (i), $u\in M_\lambda^\infty$,

$$I_\lambda^\infty(u)=I_\lambda^\infty(u)-\frac{1}{p+6}J_\lambda^\infty(u)\geqslant C\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\geqslant C\beta^2$$

$m_\lambda^\infty>0$.

引理 2 b,λ , $2<p<6$, $\forall u\in E\backslash\{0\}$, $t_u>0$, :

$$u_{t_u}=t_u u\left(\frac{x}{t_u^2}\right)\in M_\lambda^\infty\qquad I_\lambda^\infty(u_{t_u})=\max_{t>0}I_\lambda^\infty(u_t)$$

证　[2]　2.5　.

注 1　$\forall u \in M_\lambda^\infty$，$I_\lambda^\infty(u) = \max\limits_{t>0} I_\lambda^\infty(u_t)$.

引理 3　b, λ　，$2 < p < 6$，　$0 \leqslant u \in M_\lambda^\infty$，　$0 \leqslant \bar{u} \in M_\lambda^\infty \cap H_r^1(\mathbb{R}^3)$，$I_\lambda^\infty(\bar{u}) \leqslant I_\lambda^\infty(u)$，

$$H_r^1(\mathbb{R}^3) = \{u \in H^1(\mathbb{R}^3): u(x) = u(|x|)\}$$

证　$u \in M_\lambda^\infty$，　[4]　3.1.5，　$u^* \in H_r^1(\mathbb{R}^3)$，　：

$$\int_{\mathbb{R}^3} |\nabla u^*|^2 \mathrm{d}x \leqslant \int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x$$

$$\int_{\mathbb{R}^3} |u^*|^\theta \mathrm{d}x = \int_{\mathbb{R}^3} |u|^\theta \mathrm{d}x \qquad \forall \theta \in [2, 6]$$

t　，$J_\lambda^\infty\left(u^*\left(\frac{x}{t}\right)\right) > 0$.　$t > 1$　，

$$J_\lambda^\infty\left(u^*\left(\frac{x}{t}\right)\right) < t^3 J_\lambda^\infty(u) = 0$$

，　$s \in (0, 1]$，

$$J_\lambda^\infty\left(u^*\left(\frac{x}{s}\right)\right) = 0$$

$\bar{u} = u^*\left(\frac{\bullet}{s}\right)$，　$\bar{u} \in M_\lambda^\infty \cap H_r^1(\mathbb{R}^3)$，

$$I_\lambda^\infty(\bar{u}) = I_\lambda^\infty(\bar{u}) - \frac{1}{p+6} J_\lambda^\infty(\bar{u}) \leqslant I_\lambda^\infty(u) - \frac{1}{p+6} J_\lambda^\infty(u) = I_\lambda^\infty(u)$$

引理 4　b, λ　，$2 < p < 6$，　$u \in M_\lambda^\infty$，　$J_\lambda^{\infty\prime}(u) \neq 0$.

证　(　)　$u \in M_\lambda^\infty$，　$J_\lambda^{\infty\prime}(u) = 0$，　$(x, \nabla u)$　，

$$2b\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 = 0$$

$$\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x = 0$$

$u = 0$，　1(i)　.

引理 5　b, λ　，$2 < p < 6$，　$u \in M_\lambda^\infty$，$I_\lambda^\infty(u) = m_\lambda^\infty$，　u　I_λ^∞　E　.

证　$u \in M_\lambda^\infty$，$I_\lambda^\infty(u) = m_\lambda^\infty$，　4，　Lagrange　，　$\mu \in \mathbb{R}$，　$I_\lambda^{\infty\prime}(u) = \mu J_\lambda^{\infty\prime}(u)$，　u

$$I_\lambda^{\infty\prime}(u) = \mu\left[-8b\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x \Delta u + 8V_\infty u - \lambda(p+6)|u|^{p-2}u\right]$$

，

$$\mu\left[(2p-4)b\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 + (4p-8)\int_{\mathbb{R}^3} V_\infty u^2 \mathrm{d}x\right] = 0$$

$p > 2$　$u \neq 0$，　$\mu = 0$.　$I_\lambda^{\infty\prime}(u) = 0$.　u　$I_\lambda^{\infty\prime}$　E　.

定理 2 的证明　m_λ^∞　，　$\{u_n\} \subset M_\lambda^\infty$，　$I_\lambda^\infty(u_n) \to m_\lambda^\infty$.　$\{|u_n|\} \subset M_\lambda^\infty$　$I_\lambda^\infty(|u_n|) = I_\lambda^\infty(u_n)$，　$u_n \geqslant 0$.　3，　$0 \leqslant \bar{u}_n \in M_\lambda^\infty \cap H_r^1(\mathbb{R}^3)$　$I_\lambda^\infty(\bar{u}_n) \leqslant I_\lambda^\infty(u_n)$.　n　，

$$m_\lambda^\infty + 1 \geqslant I_\lambda^\infty(u_n) \geqslant I_\lambda^\infty(\bar{u}_n) \geqslant C\int_{\mathbb{R}^3} |\nabla \bar{u}_n|^2 \mathrm{d}x$$

$\bar{u}_n \in M_\lambda^\infty$，　$|\bar{u}_n|_2 \leqslant C$，　$\|\bar{u}_n\| \leqslant C$.　$0 \leqslant u \in H_r^1(\mathbb{R}^3)$，　：$H_r^1(\mathbb{R}^3)$　$\bar{u}_n \rightharpoonup u$；$L^s(\mathbb{R}^3)$　，　$\forall s \in (2, 6)$　$\bar{u}_n \to u$；　$\mathbb{R}^3$　$\bar{u}_n(x) \to u(x)$ (a.e. $x \in \mathbb{R}^3$).　$u = 0$，

$J_\lambda^\infty(\overline{u}_n) = 0$,

$$2b\left(\int_{\mathbb{R}^3} |\nabla \overline{u}_n|^2 \mathrm{d}x\right)^2 \leqslant \lim_{n\to\infty}\left[2b\left(\int_{\mathbb{R}^3} |\nabla \overline{u}_n|^2 \mathrm{d}x\right)^2 + 4\int_{\mathbb{R}^3} V_\infty \overline{u}_n{}^2 \mathrm{d}x\right] = \frac{\lambda(p+6)}{p}\int_{\mathbb{R}^3} |u|^p \mathrm{d}x = 0$$

1(i) , $u \neq 0$. 、Fatou 1 $I_\lambda^\infty(u_t) \leqslant m_\lambda^\infty$. 2, $t_u > 0$, $u_{t_u} \in M_\lambda^\infty$ $m_\lambda^\infty \leqslant I_\lambda^\infty(u_{t_u})$. u_{t_u} m_λ^∞ . 5, u_{t_u} (3) . $u_{t_u} \geqslant 0$. , $u_{t_u} > 0$, u_{t_u} (3) .

[5] Jeanjean 1.

$$X = E \qquad I_\lambda(u) = A(u) - \lambda B(u)$$

:

$$A(u) = \frac{b}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 + \frac{1}{2}\int_{\mathbb{R}^3} V(x)u^2 \mathrm{d}x \qquad B(u) = \frac{1}{p}\int_{\mathbb{R}^3} (u^+)^p \mathrm{d}x$$

$I_1(u) = I(u)$. , $\forall u \in E$, $B(u) \geqslant 0$, $\|u\| \to +\infty$, $A(u) \to +\infty$.

引理 6 $(V_1)-(V_3)$. $b > 0$ $2 < p < 6$, $\rho > 0$, $\alpha > 0$,

$$I_\lambda(u)|_{\|u\|=\rho} \geqslant \alpha \qquad \lambda \in \left[\frac{1}{2}, 1\right]$$

证 Sobolev Young , $u \in E$,

$$\begin{aligned} I_\lambda(u) \geqslant & \frac{b}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 + \frac{1}{2}\int_{\mathbb{R}^3} V(x)u^2 \mathrm{d}x - \varepsilon\int_{\mathbb{R}^3} |u|^2 \mathrm{d}x - C_\varepsilon\int_{\mathbb{R}^3} |u|^6 \mathrm{d}x \geqslant \\ & \frac{b}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 - C\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^3 \geqslant \\ & \frac{b}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2\left(1 - C\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right) \end{aligned}$$

引理 7 $(V_1)-(V_3)$, $b > 0$, $2 < p < 6$. :

(i) $\forall \lambda \in \left[\frac{1}{2}, 1\right]$, $\rho > 0$, $v_2 \in E$ $\|v_2\| > \rho$, $I_\lambda(v_2) < 0$;

(ii) $\forall \lambda \in \left[\frac{1}{2}, 1\right]$,

$$c_\lambda = \inf_{\gamma \in \Gamma} \max_{t \in [0, 1]} I_\lambda(\gamma(t)) > \max\{I_\lambda(0), I_\lambda(v_2)\}$$

$$\Gamma = \{\gamma \in C([0, 1], E): \gamma(0) = 0, \gamma(1) = v_2\}$$

证 (i) $0 < u \in E$, $u_t = tu\left(\frac{x}{t^2}\right)$, $t > 0$.

$$\begin{aligned} I_\lambda(u_t) = & \frac{bt^8}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 + \frac{t^8}{2}\int_{\mathbb{R}^3} V(t^2 x)\, u^2 \mathrm{d}x - \frac{\lambda t^{p+6}}{p}\int_{\mathbb{R}^3} u^p \mathrm{d}x \leqslant \\ & \frac{bt^8}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)^2 + \frac{t^8}{2}\int_{\mathbb{R}^3} V_\infty u^2 \mathrm{d}x - \frac{t^{p+6}}{2p}\int_{\mathbb{R}^3} u^p \mathrm{d}x \end{aligned}$$

$p > 2$, $t \to +\infty$, $I_\lambda(u_t) \to -\infty$. $t_0 > 0$, $\|u_{t_0}\| > \rho$, $\forall \lambda \in \left[\frac{1}{2}, 1\right]$, $I_\lambda(u_{t_0}) < 0$. $v_2 = u_{t_0}$, (i) .

(ii) (i)、 6 c_λ , $c_{\frac{1}{2}} \geqslant c_\lambda \geqslant c_1 \geqslant \alpha > 0$. $I_\lambda(0) = 0$ $I_\lambda(v_2) < 0$, (ii) .

引理 8 $(V_1)-(V_3)$, $b > 0$ $2 < p < 6$, $\forall \lambda \in \left[\frac{1}{2}, 1\right]$, $m_\lambda^\infty > c_\lambda$.

证 $0 < u_\lambda^\infty \in E$ m_λ^∞ . 7 , $t_0 > 0$ $I_\lambda\left(t_0 u_\lambda^\infty\left(\frac{x}{t_0^2}\right)\right) < 0$.

$$\gamma(t)=\begin{cases}0 & t=0\\ tt_0u_\lambda^\infty\left(\dfrac{x}{(tt_0)^2}\right) & t\in(0,1]\end{cases}$$

$\gamma(t)$ $[0,1]$. $\gamma\in\Gamma$. (V_2), $\forall\lambda\in\left[\frac{1}{2},1\right]$,

$$m_\lambda^\infty=I_\lambda^\infty(u_\lambda^\infty)=\max_{t>0}I_\lambda^\infty\left(tu_\lambda^\infty\left(\frac{x}{t^2}\right)\right)>$$
$$\max_{t>0}I_\lambda\left(tu_\lambda^\infty\left(\frac{x}{t^2}\right)\right)\geqslant\max_{t\in(0,1]}I_\lambda\left(tt_0u_\lambda^\infty\left(\frac{x}{(tt_0)^2}\right)\right)=$$
$$\max_{t\in[0,1]}I_\lambda(\gamma(t))\geqslant c_\lambda$$

引理 9 $(V_1)-(V_3)$ ，$\{u_n\}\subset E$ I_λ $(PS)_{c_\lambda}$. $b>0$，$2<p<6$ ， $0\leqslant u\in E$， E $u_n\to u$.

证 [2] 3.4， $l=0$. $l\geqslant 1$， $\zeta_\lambda^{\infty\prime}(v_j)=0$， ：

$$\langle\zeta_\lambda^{\infty\prime}(v_j),v_j\rangle=0\qquad\langle\zeta_\lambda^{\infty\prime}(v_j),(x,\nabla v_j)\rangle=0$$

$$d=\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x+\sum_{j=1}^{l}\int_{\mathbb{R}^3}|\nabla v_j|^2\mathrm{d}x$$

$$0=\langle\zeta_\lambda^{\infty\prime}(v_j),v_j\rangle+2\langle\zeta_\lambda^{\infty\prime}(v_j),(x,\nabla v_j)\rangle\geqslant$$
$$2b\left(\int_{\mathbb{R}^3}|\nabla v_j|^2\mathrm{d}x\right)^2+4\int_{\mathbb{R}^3}V_\infty v_j^2\mathrm{d}x-\frac{\lambda(p+6)}{p}\int_{\mathbb{R}^3}|v_j|^p\mathrm{d}x=J_\lambda^\infty(v_j)$$

$t_j\in(0,1]$，

$$t_jv_j\left(\frac{x}{t_j^2}\right)\in M_\lambda^\infty$$

$$d=\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x+\sum_{j=1}^{l}\int_{\mathbb{R}^3}|\nabla v_j|^2\mathrm{d}x$$

$$m_\lambda^\infty+\frac{bd}{4}\int_{\mathbb{R}^3}|\nabla v_j|^2\mathrm{d}x\leqslant\zeta_\lambda^\infty(v_j)-\frac{\langle\zeta_\lambda^{\infty\prime}(v_j),v_j\rangle+2\langle\zeta_\lambda^{\infty\prime}(v_j),(x,\nabla v_j)\rangle}{p+6}=\zeta_\lambda^\infty(v_j)$$

(V_1)，

$$\zeta_\lambda(u)\geqslant\frac{bd}{4}\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x$$

$$c_\lambda+\frac{bd^2}{4}=\zeta_\lambda(u)+\sum_{j=1}^{l}\zeta_\lambda^\infty(v_j)\geqslant$$
$$\frac{bd}{4}\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x+lm_\lambda^\infty+\frac{bd}{4}\sum_{j=1}^{l}\int_{\mathbb{R}^3}|\nabla v_j|^2\mathrm{d}x=$$
$$lm_\lambda^\infty+\frac{bd^2}{4}\geqslant m_\lambda^\infty+\frac{bd^2}{4}$$

8 ， $l=0$.

引理 10 $(V_1)-(V_3)$. $b>0$ $2<p<6$， $\forall\lambda\in\left[\frac{1}{2},1\right]$， $I(u)$ E

.

证 Jeanjean 、 6 7, $\forall \lambda \in \left[\frac{1}{2}, 1\right]$, I_λ E $(PS)_{c_\lambda}$.
$u^- = \max\{-u, 0\}$, , E $u_n^- \to 0$, :

$$c_\lambda = I_\lambda(u_n) + o(1) = I_\lambda(u_n^+) + o(1)$$

$$0 = \langle I'_\lambda(u_n), \varphi\rangle + o(1) = \langle I'_\lambda(u_n^+), \varphi\rangle + o(1)$$

$\varphi \in E$, $\|\varphi\| = 1$. $u_n \geqslant 0$. 9, , $0 \leqslant u \in E$, E $u_n \to u$. $I_\lambda(u_n) \to I_\lambda(u) = c_\lambda$, E^* $I'_\lambda(u_n) \to I'_\lambda(u) = 0$.

定理 1 的证明 10, $\forall \lambda_j \in \left[\frac{1}{2}, 1\right]$, $u_j \in E$, $\lambda_j \to 1^-$, $I_{\lambda_j}(u_j) = c_{\lambda_j}$ $I'_{\lambda_j}(u_j) = 0$. :

$$\langle I'_{\lambda_j}(u_j), u_j\rangle = 0 \qquad \langle I'_{\lambda_j}(u_j), (x, \nabla u_j)\rangle = 0$$

:

$$A = b\left(\int_{\mathbb{R}^3} |\nabla u_j|^2 \mathrm{d}x\right)^2$$

$$B = \int_{\mathbb{R}^3} V(x) u_j^2 \mathrm{d}x \qquad C = \lambda_j \int_{\mathbb{R}^3} u_j^p \mathrm{d}x$$

$$\begin{cases} A + B - C = 0 \\ \dfrac{A}{2} + \dfrac{3B}{2} + \dfrac{1}{2}\displaystyle\int_{\mathbb{R}^3} (\nabla V(x), x) u_j^2 \mathrm{d}x - \dfrac{3}{p} C = 0 \end{cases}$$

(V_1) ,

$$\begin{aligned} c_{\lambda_j} = I_{\lambda_j}(u_j) = {} & \\ & \frac{A}{4} + \frac{B}{2} - \frac{1}{p} \cdot \frac{p}{4p-6}(A+B) - \frac{1}{p} \cdot \frac{3p-6}{4p-6} \cdot \frac{p}{3}\left[\frac{A}{2} + \frac{3B}{2} + \frac{1}{2}\int_{\mathbb{R}^3} (\nabla V(x), x) u_j^2 \mathrm{d}x\right] = \\ & \frac{(p-2)b}{4(p+6)}\left(\int_{\mathbb{R}^3} |\nabla u_j|^2 \mathrm{d}x\right)^2 + \frac{p-2}{2(p+6)}\int_{\mathbb{R}^3} V(x) u_j^2 \mathrm{d}x - \frac{1}{p+6}\int_{\mathbb{R}^3} (\nabla V(x), x) u_j^2 \mathrm{d}x \geqslant \\ & \frac{(p-2)b}{4(p+6)}\left(\int_{\mathbb{R}^3} |\nabla u_j|^2 \mathrm{d}x\right)^2 \end{aligned}$$

$C > 0$, $\|u_j\|_{D^{1,2}(\mathbb{R}^3)} \leqslant C$.

$$\langle I'_{\lambda_j}(u_j), u_j\rangle = 0$$

Young Sobolev ,

$$C\int_{\mathbb{R}^3} u_j^2 \mathrm{d}x \leqslant b\left(\int_{\mathbb{R}^3} |\nabla u_j|^2 \mathrm{d}x\right)^2 + \int_{\mathbb{R}^3} V(x) u_j^2 \mathrm{d}x \leqslant$$

$$\int_{\mathbb{R}^3} u_j^p \mathrm{d}x \leqslant \frac{C}{2}\int_{\mathbb{R}^3} u_j^2 \mathrm{d}x + C\int_{\mathbb{R}^3} u_j^6 \mathrm{d}x$$

$\frac{6-p}{4} < 1$, $\{u_j\}$ $L^2(\mathbb{R}^3)$, $\{u_j\}$ E .

$$I(u_j) = I_{\lambda_j}(u_j) + o(1) = c_{\lambda_j} + o(1) = c + o(1)$$

E^*

$$I'(u_j) = I'_{\lambda_j}(u_j) + o(1) = o(1)$$

10, $0 \leqslant u \in E$, E $u_j \to u$, $I(u_j) \to I(u) = c$, E^* $I'(u_j) \to I'(u) = 0$, u (1) . (1) ,

$$\pi = \inf_{u \in \prod} I(u)$$

$$\prod = \{u \in E \backslash \{0\} : I'(u) = 0, u \geqslant 0\}$$

$\prod \neq \emptyset$,　$0 \leqslant u_n \in E$,　$I'(u_n) = 0$　$I(u_n) \to \pi$.　, $\{u_n\}$　.　9,
$0 \leqslant u \in E$　$u_n \to u$,　:

$$I(u_n) \to I(u) = \pi \qquad I'(u_n) \to I'(u) = 0$$

,　$u > 0$.

参考文献:

[1] LIU Z S, GUO S J. Existence of Positive Ground State Solutions for Kirchhoff Type Problems [J]. Nonlinear Anal, 2015, 120: 1-13.

[2] LI G B, YE H Y. Existence of Positive Ground State Solutions for the Nonlinear Kirchhoff Type Equations in $\mathbb{R}^3$ [J]. J Differential Equations 2013, 257(2): 566-600.

[3] XIE Q L, WU X P, TANG C L. Existence and Multiplicity of Solutions for Kirchhoff Type Problem with Critical Exponent [J]. Communications on Pure and Applied Anylisis, 2013, 12(6): 2773-2786.

[4] LIONS P L. The Concentration-Compactness Principle in the Calculus of Variations. The Locally Compact Case, Part I [J]. Analyse Nonlinéaire, 1984, 1(2): 109-145.

[5] JEANJEAN L, TANAKA K. A Positive Solution for a Nonlinear Schrödinger Equation on $\mathbb{R}^N$ [J]. Indiana Univ Math, 2005, 54(2): 443-464.

[6] ,　.　Kirchhoff　[J].　(　), 2015, 37(2): 60-63.

Positive Ground State Solutions for Degenerate Kirchhoff-Type Equation

LIU Ting-ting,　SHANG Yan-ying

School of Mathematics and Statistics, Southwest University, Chongqing 400715, China

Abstract: In this paper, using the variational methods, we deal with the existence of positive state solutions for Degenerate Kirchhoff-type problem with nonlinear team in $\mathbb{R}^3$:

$$\begin{cases} -\left(b\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\Delta u + V(x)u = |u|^{p-2}u & x \in \mathbb{R}^3 \\ u \in H^1(\mathbb{R}^3),\ u > 0 & x \in \mathbb{R}^3 \end{cases}$$

Key words: Kirchhoff-type equation; state solutions; variational method; Pohozaev equality

责任编辑　廖　坤