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# The Sixth Graders' Conceptual Understanding about Computational Estimation in Primary School①

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**Abstract**: Using three self-designed tasks, this paper investigated the sixth graders' conceptual understanding about computational estimation in Chinese primary school. 69 children randomly sampling from an ordinary primary school participated in this study. The results showed that ① primary school children gained much more conceptual knowledge and procedural knowledge than conditional knowledge about computational estimation; ② there was a significant correlation in the level of mastery between conceptual knowledge and conditional knowledge; and ③ only the level of conceptual knowledge contributed significantly to the difference of computational estimation performance between the high-level group and the low-level group. The results and their implications for the instruction of mathematics in primary school were also discussed.

**Key words**: conceptual understanding; computational estimation; primary school children

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In everyday life, many mathematical situations call for producing an estimate. Computational estimation is just one of most common mathematical estimation activities, which requires finding an estimate of the result of a computation by performing some mental computation on approximations of the original numbers. The data from NAEP conducted by Carpenter et al offered ample evidence that students lacked basic and necessary estimation skills for carrying out many real mathematical problems which need to be estimated[1-2].

In the past three decades, computational estimation has attracted interesting attention from both educators[3-6] and cognitive psychologists[7-12]. Many aspects of computational estimation have been investigated, such as components, types of strategies and their development, and instruction. However, individuals' conceptual understanding about computational estimation is not well understood. Delineating this issue is fundamental to understanding how computational estimation activity occurs. The previous related literature on strategic behavior shows that students often do not use their strategic knowledge to the ful-

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lest[13]. Many students just guess when confronting a computational problem to estimate instead of using any effective strategy, which tightly ties up students' understandings on computational estimation. In a study with 12 to 14-year-olds, Morgan (1988) found that most of the children she interviewed did not have clear conception of the purpose or the nature of computational estimation[14]. They seemed to consider estimation to be an inferior alternative to exact computation, and described estimation as "guessing". In fact, some of their estimates were given in a "non-rounded" form because they believed that the estimate should look as though it could be the exact answer. Some other investigators also have found indications that young children do not understand what estimation means[15-16]. Why this discrepancy occurs between strategic knowledge and the implementation of that knowledge in a specific task situation is a very important issue. Some authors link the "production deficiency"[17] to the development of children's metacognitive abilities[18-19]. Not only do children need to know when and how a certain strategy can be implemented[20], but they need to recognize the value of the strategy[21] and its relative usefulness compared to other strategies[22].

In the present study, conceptual understanding is supposed to include three types of general knowledge, that is conceptual knowledge, procedural knowledge and conditional knowledge. When it comes to computational estimation, conceptual understanding is concerned with the following aspects: the meanings of computational estimation and related concepts belongs to conceptual knowledge, how to execute computational estimation strategies and related skills belongs to procedural knowledge, when and why to execute computational estimation and its special strategy belongs to conditional knowledge. If any aspect of conceptual understanding is absent, the quality and efficiency of problem solving will be impacted seriously. The main purpose of the present study is just to investigate the role of primary school pupils' understanding about computational estimation and its skills on their implements. We predict that children with good understanding about computational estimation will perform higher achievement than those with poor understanding.

# 1 Method

## 1.1 Participants

69 sixth-graders sampling from an ordinary primary school in Chongqing City participated in this research, including 40 boys and 29 girls. The average age of these participants is 11.98 years old and the standard deviation is 0.57.

## 1.2 Materials

Some self-developed tasks to evaluate individuals' conceptual understanding and a computational estimation test for group were used.

There were three types of problems in the task to evaluate individuals' conceptual understanding. The first one was concept-explaining task (CET), which was used to judge declarative knowledge about the concept of computational estimation. An imaginary scene designed to meet this goal was displayed as follows. "If your mathematical teacher wants you to teach a lesson about mathematics to your classmates in place of him, you need to review the knowledge about computational estimation you have learned. They gave some questions about this part. You can answer these questions to explain your meaning by examples". In this supposed scene, all participants were asked to answer these questions which involved the definition of computational estimation, concerned knowledge, skills and the situations needed to use, et al. The second one was situation-judging task (SJT), which was to evaluate whether an individual knew those situations that need to use computational estimation. According to these answers of this type problem, we can assess their conditional knowledge about this method. The third one was strategy-choosing task

(SCT). The subjects were asked to choose the appropriate method for the test item about computational estimation to assess their procedural knowledge.

### 1.3 Procedures

A within-subject design was used in this research. Under the experimenters' directions, each participant finished three tasks to evaluate his conceptual understanding and received a group test about computational estimation. Time was limitless when finishing evaluating tasks, but the group test about computational estimation was limited within five minutes (the length of time is confirmed according to a previously small-scale preliminary test). The request about computational estimation test was to give an estimate for each item.

### 1.4 Coding

For CET, the performance was scaled at a five-point scale. If an individual possessed no knowledge about computational estimation, we coded this case zero point. If he owned some of knowledge with very superficial understandings, we coded it one point. If he owned some of knowledge with moderate understandings, we coded it two points. If he owned much knowledge with moderate understanding, we coded it three points. Only a participant owned much knowledge with very deep understandings, we coded his performance 4 points. According to the above standards drew out by the researchers, two independent rater valued all participants' performance. The result showed that the consistency coefficient between two raters was 0.83 ( $p<0.01$). For SJT, the performance was scaled at a two-point scale. If the participant's judge was right and he also gave proper reasons, we coded his performance two points. Compared to this, when his judge was wrong or showed a wrong reason, we coded his performance zero point. As far as SCT, we scored according to the number of appropriate strategies selected by subjects.

## 2 Results

### 2.1 The Status Quo of Children's Conceptual Understanding about Computational Estimation

Table 1 showed the mean and standard deviation of children's performance in three types of tasks. A t test with paired groups was used to investigate the differences among different tasks.

As can be seen in table 1, there is a significant difference when children master different contents of the concept of computational estimation. On the whole, the performances at CET and SCT are significantly better than at SJT. The performance at CET is also better than at SCT. In the present study, we investigated declarative knowledge and procedural knowledge at CET and SCT respectively, and SJT was used to evaluate the conditional knowledge. So we can draw a conclusion that primary school children gained much more conceptual knowledge and procedural knowledge than conditional knowledge about computational estimation.

In addition, correlation analysis showed there was a strongly positive correlation between the scores of SJT and CET ($\gamma=0.46$, $p<0.01$), but no such a similar relation existed between SJT and SCT. At the same time, we also didn't find a significant correlation between CET and SCT. This case indicates that whether children can judge the necessity of using computational estimation depends on the quantity and quality of conceptual knowledge they grasped. But the judge ability can't become the sufficient condition to use computational estimation strategies accurately. Synchronously, the amounts of conceptual knowledge had no inevitable relation with whether a person exerts computational estimation strategies accurately. So we can conclude that the appropriate utility of strategy about computational estimation can't change with the amount of conceptual knowledge but relies on many other factors. We also can't eliminate the possibility that the level of conceptual knowledge about computational estimation hasn't differentiated at all.

**Table 1 Children's performance differences in conceptual understanding tasks**

| | CET | SJT | SCT | $T_{CET\text{-}SJT}$ | $T_{CET\text{-}SCT}$ | $T_{SJT\text{-}SCT}$ |
|---|---|---|---|---|---|---|
| *M* | 2.21 | 0.74 | 1.54 | 8.78** | 3.93** | 7.24** |
| SD | 1.01 | 0.55 | 0.70 | | | |

Note: CET means concept explaining task, SJT means situation judging task, SCT means strategy choosing task. ** means $p<0.01$

## 2.2 Effect of Conceptual Understanding on Computational Estimation Performance

According to the mean of scores of individuals' performances at the whole conceptual understanding task and each subtask, all participants were divided into the high-level group (HG) and low-level group (LG). A t test with independent group was used to evaluate the difference of performance at computational estimation between different groups in all tasks (table 2).

The results showed that there was a significant difference between HG and LG at CET ($t=2.21$, $p<0.05$), and we didn't find any significant difference in any other tasks. It indicates that the amount of individuals' conceptual knowledge about computational estimation during the primary school stage may affect their performances. It isn't intricate to understand this result. Because the primary school children in China doesn't get in touch with computational estimation until third grade. In addition, according to the actual curriculum standard issued by Ministry of Education of China, the content about computational estimation is a voluntary part to teach in mathematics for primary school. So many mathematical teachers think it dispensable. In their instructions, they don't teach many skills about computational estimation, but teach some related conceptual knowledge. Just for this reason, students spend few time and efforts learning computational estimation, and they can't master much conditional knowledge of effective strategies and those circumstances in which they should use these strategies. Herein, the phenomenon means that there only exists a significant difference at CET. We can conclude that it is urgent for Chinese teacher to strengthen the instruction about computational estimation strategies to enhance pupils' practical ability. After all, pupils can get some concrete experiences about computational estimation. But if we expect they also can accumulate procedural knowledge and conditional knowledge spontaneously, it is a bit impossible.

**Table 2 Comparison about children's computational estimation performance in conceptual understanding tasks between different groups**

| | CET | | SJT | | SCT | | Total | |
|---|---|---|---|---|---|---|---|---|
| | HG | LG | HG | LG | HG | LG | HG | LG |
| *M* | 66.23 | 56.33 | 62.78 | 58.79 | 58.87 | 61.22 | 59.12 | 58.37 |
| *SD* | 10.27 | 13.98 | 12.11 | 14.17 | 15.65 | 11.47 | 13.43 | 14.55 |
| *T* | 2.21* | | 0.75 | | −0.50 | | 0.12 | |

Note: HG means high-level group, LG means low-level group. * means $p<0.05$

Because we found there was a considerable close connection between children's amounts of conceptual knowledge about computational estimation and their performances, a t test with independent group was used to investigate the difference of performance between HG and LG about conceptual knowledge (table 3). The results showed that there wasn't a significant difference among different types of items. It indicates that conceptual knowledge affects all tasks about computational estimation, and the size of effect is similar among different tasks.

**Table 3 Performance differences between children with different mastery level of conceptual knowledge**

| | Whole Number | | Decimal | | Fraction | |
|---|---|---|---|---|---|---|
| | HG | LG | HG | LG | HG | LG |
| *M* | 2.16 | 1.93 | 2.01 | 1.77 | 1.38 | 0.98 |
| *SD* | 0.51 | 0.57 | 0.54 | 0.78 | 0.53 | 0.58 |
| *T* | 1.18 | | 1.27 | | 1.80 | |

## 3 Discussions

In this research, we found that in computational estimation tasks, the mastery level about conceptual knowledge and procedural knowledge was significantly better than that of conditional knowledge. It has an important meaning for us to teach computational estimation in primary school. This result indicates the biggest limitation in present instruction about computational estimation is the absence of teaching conditional knowledge. As computational estimation is gradually important for mathematics instruction in primary school, the vast teachers must be aware of this problem ahead of schedule and take actions to overcome it effectively.

In fact, it exists in many subjects to teach conditional knowledge with deficient efforts, and it is also a difficulty for strategy instruction. In many circumstances, students don't know how to make use of the knowledge learned from subject matter instructions. As a result, it is easy for pupils to transit knowledge into a renitent state. Under this state, knowledge can be retrieved only in a very limited background, such as in a similar context of knowledge acquisition. Many researches have shown that even strategies mastered by students can also be in a renitent state, unless the teacher asked students to use definitely [23]. Thus, to avoid the appearance of renitent knowledge, it is necessary for teachers to request students to integrate the basic knowledge and strategies with their conditions when they deposit knowledge into their brains. Just in this case, students can acquire conditional knowledge. So at the moment of teaching students' conceptual knowledge and procedural knowledge about computational estimation, teachers must ask them to make clear the conditions of knowledge execution. After all, it is an important mark for an individual's development of ability to retrieve and apply exactly concerned knowledge to meet problems. In this research, we also found that there was a significantly strong correlation between the primary school children's performances and their mastery level about conceptual knowledge. It is an evidence to carry out the conditionalization of knowledge in the instruction of computational estimation. To enhance the retrievability of conceptual knowledge and procedural knowledge about computational estimation to solve practical problem, it is also for teachers to prompt students to consider the condition to apply in a background outside the classroom when they present these knowledge. The effect of this measure has been confirmed by many researches in other fields [24]. It still needs further studies to test the effect in the instruction of computational estimation.

In this research, we also found that what had a close association with children's performances about computational estimation was the quantity and quality of their conceptual knowledge. But there was no obvious relation between whether they can judge the situations to use computational estimation and whether they can actually choose those effective strategies. We suppose that it is possible for primary school children to accumulate insufficient procedural knowledge and conditional knowledge in nature. They can't show any significant differences in these types of knowledge. That is, there exists a floor effect for children to master these two kinds of knowledge. But the same case doesn't occur at the mastery of conceptual knowledge. Because before the formal contact to computational estimation, children have formed some na?ve beliefs about this concept and they can show significantly individual differences. Because different cognitive styles maybe affect adults' performance in computational estimation in virtue of restricting the transfer and application of different types of knowledge [25], We firmly believe that elder children may show significant differences potentially as their procedural knowledge and conditional knowledge become more and more abundant. Another possible theoretical explanation is problem representation, which has been confirmed to obviously confine the efficiency of problem solving not only with domain-general task [26] but with domain-specific task [27]. Of course, all the above-mentioned inferences should be verified by further researches in the future.

## 4 Conclusion

According to the above-mentioned analysis, we can draw some conclusions as follows. With regard to conceptual understandings about computational estimation, it is obviously better for primary school children to master conceptual knowledge and procedural knowledge than to master conditional knowledge. Whether children can judge accurately the necessity to use computational estimation relies on the quantity and quality of their conceptual knowledge. Only in concept explanation task, there was a significant difference in performances about computational estimation between HG and LG.

**References:**

[1] Carpenter T P, Coburn T G, Reys R E, et al. Notes From National Assessment: Estimation [J]. Arithmetic Teacher, 1976, 23 (4): 296－302.

[2] Carpenter T P, Corbitt M K, Kepner H S, et al. National Assessment: A Perspective of Students' Mastery of Basic Mathematics Skills [C]//M. M. Lindquist (Ed.), Selected issues in mathematics education. Reston, VA: NCTM, 1980: 215－257.

[3] Levine D R. Strategy use and Estimation Ability of College Students [J]. Journal for Research in Mathematics Education, 1982, 13(5): 350－359.

[4] Reys R E, Rybolt J F, Bestgen B J, et al. Processes Used by Good Computational Estimators [J]. Journal for Research in Mathematics Education, 1982, 13(2): 183－201.

[5] Rubenstein R N. Computational Estimation and Related Mathematical Skills [J]. Journal for Research in Mathematics Education, 1985, 16(2): 106－119.

[6] Sowder J T. Estimation and Related Topics [C]//D. A. Grouws (Ed.), Handbook of research in mathematics teaching and learning. NY: Macmillan, 1992: 371－389.

[7] Dehaene S, Cohen L. Two Mental Calculation Systems: A Case Study of Severe Acalculia With Preserved Approximation [J]. Neuropsychologia, 1991, 29(11): 1045－1054.

[8] Dehaene S, Spelke E, Pinel P, et al. Sources of Mathematical Thinking: Behavioral and Brain-imaging Evidence [J]. Science, 1999, 284(5416): 970－973.

[9] Dowker A D, Flood A, Griffiths H, et al. Estimation Strategies of Four Groups [J]. Mathematical Cognition, 1996, 2 (2): 113－135.

[10] LeFevre J, Greenham S L, Waheed N. The Development of Procedural and Conceptual Knowledge in Computational Estimation [J]. Cognition and Instruction, 1993, 11(2): 95－132.

[11] Lemaire P, Lacacheur M. Children's Strategies in Computational Estimation [J]. Journal of Experimental Child Psychology, 2002, 82(4): 281－304.

[12] Warrington E K. The Fractionation of Arithmetic Skills: a Single Case Study [J]. Quarterly Journal of Experimental Psychology, 1982, 34A (1): 31－51.

[13] Garcia T, Pintrich P R. Regulating Motivation and Cognition in the Classroom: The Role of Self-schemas and Self-regulatory Strategies [C]//D. H. Schunk, B. J. Zimmerman (Eds.), Self-regulation of learning and performance: issues and educational applications. Hillsdale, NJ: Erlbaum, 1994: 127－154.

[14] Morgan C. A Study of Estimation by Secondary School Children [D]. Unpublished MSc dissertation, Institute of Education, London, 1988.

[15] Clayton J G. Estimation [J]. Mathematics Teaching, 1988, 125: 18－19.

[16] Sowder J T, Wheeler M M. The Development of Concepts and Strategies Used in Computational Estimation [J]. Journal for Research in Mathematics Education, 1989, 20(2): 130－146.

[17] Flavell J H. Developmental Studies of Mediated Memory [C]//H. W. Reese, L. P. Lipsitt (Eds.), Advances in child development and behavior (Vol. 5). New York: Academic Press, 1970: 181－211.

[18] Borkowski J G, Kurtz B E. Metacognition and Executive Control [C]//J. G. Borkowski, J. D. Day (Eds.), Cognition and intelligence in special children: Comparative approaches to retardation, learning disabilities, and giftedness. Nor-

wood, N. J.: Ablex Publishing Corporation, 1987: 123－152.

[19] Flavell J H, Wellman H M. Metamemory [C]//R. V. Kail, J. W. Hagen (Eds.), Perspectives on the development of memory and cognition. NY: Macmillan, 1977: 243－275.

[20] Borkowski J G. Signs of Intelligence: Strategy Generalization and Metacognition [C]//S. R. Yussen (Ed.), the growth of reflection in children. Orlando, FL: Academic Press, 1985: 106－144.

[21] Kurtz B E, Weinert F E. Metamemory, Memory Performance, and Causal Attribution in Gifted and Average Children [J]. Journal of Experimental Child Psychology, 1989, 48(1): 45－61.

[22] Fabricius W V, Hagen J W. Use of Causal Attributions About Recall Performance to Assess Metamemory and Predict Strategic Memory Behavior in Young Children [J]. Developmental Psychology, 1984, 20(5): 975－987.

[23] Brown A L, Day, J D. Macrorules for Summarizing Text: The Development of Expertise [J]. Journal of Verbal Learning and Verbal Behavior, 1983, 22(1): 1－14.

[24] Branford J, Sherwood R, Vye N, et al. Teaching thinking and problem solving [J]. American Psychologist, 1986, 41 (10): 1078－1089.

[25] 张云仙，司继伟. 大学生的认知方式对其估算能力的影响 [J]. 西南师范大学学报(自然科学版)，2007，32(3)：168－171.

[26] 史滋福，邱　江，张　卫，等. 表征方式与数理图式对解决四卡问题的影响 [J]. 西南师范大学学报(自然科学版)，2006，31(5)：177－181.

[27] 徐继红，司继伟. 不同干扰刺激对成人数量估计的影响 [J]. 心理科学，2008，31(1)：195－199.

# 小学六年级儿童估算的概念理解

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**摘要：**采用三种自行设计的任务考察了小学六年级儿童对估算的概念理解. 从一所普通小学随机抽取的 69 名被试参加了本实验. 结果表明：①小学儿童获取的有关估算的概念性知识与程序性知识明显多于条件性知识；②概念性知识与条件性知识的掌握水平之间存在显著正相关；③只有概念性知识的掌握水平明显制约了高、低组儿童的估算表现差异. 文中对上述结果及其在小学数学中的教学含义进行了讨论.

**关键词：**概念理解；估算；小学儿童

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