文章编号：1000－5471(2007)06－0052－04

Study on the Reliability of Dependence-parts Vote System Based on Copula Functions①

YI Wen-de[1]，　WEI Gui-wu[2]

1. Dept. of Mathematics and Computer Science, Chongqing University of Arts and Sciences, Yongchuan Chongqing 402160, China;
2. Dept. of Economics and Management, Chongqing Univ. of Arts and Sciences, Chongqing 402160, China

Abstract: In reliability theory, the independence is a particular state of the dependence. Reliability mathematics carries through the reliability researches of system that almost suppose that the parts are in the condition of inter-independence. This paper is based on the knowledge of the Copula function to study the reliability of vote system that consists of dependence parts and the relation between the system and copula and to show that how the dependence of parts made the reliability measure of the system.

Key words: reliability; vote system; copula; dependence; independence

CLC number: 0213. 2　　**Document code**: A

The reliability theory has some drawbacks: the theorem of independency-hypothesis is only for a particular condition of complete non-correlation of parts[1], and the theorem of weak circular only for line-correlation[2]. It is generally assumed that components of system are independence to study the life and quantity measurement of system in reliability mathematics, but not deal with the dependency of components. As we know well, the dependent relation between components of system is not simply restricted in the linear correlation and may be a non-linear correlation.

It's well known, the correlation coefficient ρ is described in traditional probability and statistics is useful only restricted in linear correlation and independency regime, but invalidity in non-linear correlation regime. For example, let $X \sim N(0, 1)$, $Y = X^2$, there is obviously a functional relation between X and Y, but their correlation coefficient $\rho = 0$. Sklar[4] has given an expression for a multivariate distribution in terms of a copula and their marginal distributions, where the copula depicts the dependent structure of random variables.

In this paper, we investigate the relation between the copula functions and the reliability measurement of vote systems based on the reliability mathematics and the copulas.

Let $X_1, X_2, \cdots, X_n$ be dependent lifetime variables of components, then the reliability of the series of n components is given by[5-6]

$$\begin{aligned} R(t) &= P\{\min(X_1, X_2, \cdots, X_n) > t\} \\ &= P\{X_1 > t, X_2 > t, \cdots, X_n > t\} \\ &= \sum \mathrm{sgn}(\mathrm{F}) C^{(n)}(F) \end{aligned}$$

① 收稿日期：2007－04－17
作者简介：易文德(1965－)，男，江西宜春人，副教授，博士研究生，主要从事概率统计及其应用研究.

Where $F=(F_1, F_2, \cdots, F_n)$, $F_i = F_i(t)$ or 1, $F_i(\infty) = 1$.

$$\text{sgn}(F) = \begin{cases} 1, & if\ F_i = F_i(t)\ for\ an\ even\ number\ of\ i's, \\ -1, & if\ F_i = F_i(t)\ for\ an\ odd\ number\ of\ i's, \end{cases}$$

The reliability measure of parallel connection system consist of n dependent components is given by

$$R(t) = p\{\max(X_1, \cdots, X_n) > t\} = 1 - P\{\max(X_1, \cdots, X_n) \leqslant t\}$$
$$= 1 - C(F_1(t), \cdots, F_n(t))$$

1 Reliability analysis of dependent-parts vote system

A system consisted of n components can works well only if k or more components of n components are good ($1 \leqslant k \leqslant n$), i. e. the system will be a hazard if the number of failure components is larger or equal to $n-k+1$. The system is called as a vote system of $k$components chosen from n components, denotes as $k/n(G)$. See Fig. 1.

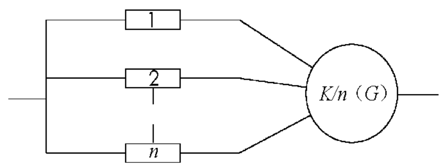

Fig 1 Vote system

Suppose $X_1, X_2, \cdots, X_n$ are life variables and every component is new at time $t=0$, and works at the same time. First, consider n components are inter-independent and i. i. d. Let the reliability measurement of every part be $R_0(t)$, then the reliability measurement of the system is given by

$$R(t) = \sum_{j=k}^{n} C_n^j \cdot P\{X_{j+1}, \cdots, X_n \leqslant t \leqslant X_1, \cdots, X_j\}$$
$$= \sum_{j=k}^{n} C_n^j \cdot R_0^j(t)[1-R_0(t)]^{n-j}$$

where C_n^j is combination number randomly chosen j from n elements[3].

Now consider that the components in system are dependency each other. The dependency relation may be either linear correlation or non-linear correlation. Let X_i be the lifetime variable of the ith part with the distribution function $F_i(t) = P\{X_i \leqslant t\}$. Then the reliability measurement is given by

$$R_i(t) = P\{X_i > t\} = 1 - F_i(t),\ i = 1, 2, \cdots, n.$$

The joint distribution of $X_1, X_2, \cdots, X_n$ is given by

$$H(x_1, x_2, \cdots, x_n) = P\{X_1 \leqslant x_1, X_2 \leqslant x_2, \cdots, X_n \leqslant x_n\} \quad (1)$$

According to Sklar's theorem, there exists a n-copula function $C^{(n)}$, such that

$$H(x_1, x_2, \cdots, x_n) = C^{(n)}(F_1(x_1), F_2(x_2), \cdots, F_n(x_n)). \quad (2)$$

Let $X_m^{(1)}, X_m^{(2)}, \cdots, X_m^{(j)}$ denote the life of j parts randomly chosen from $X_1, X_2, \cdots, X_n$ in the m'th, where $m = 1, 2, \cdots, C_n^j$. The univariate distribution function of every $X_m^{(1)}, X_m^{(2)}, \cdots, X_m^{(j)}$ is the same that of the correspond to assumption respectively. For uniform signs, the distributions of their lives denote $F_m^{(1)}, F_m^{(2)}, \cdots, F_m^{(j)}$ respectively, and the joint distribution of them is a j variants marginal distribution obtained from (1), denoted by $H_m^{(j)}(x_m^{(1)}, x_m^{(2)}, \cdots, X_m^{(j)})$, and their copula is a marginal copula obtained from (2) which given by

$$C_m^{(j)}(F_m^{(1)}(x_m^{(1)}), F_m^{(2)}(x_m^{(2)}), \cdots, F_m^{(j)}(x_m^{(j)}))$$

Apparently, $H_m^{(j)}(x_m^{(1)}, x_m^{(2)}, \cdots, X_m^{(j)}) = C_m^{(j)}(F_m^{(1)}(x_m^{(1)}), F_m^{(2)}(x_m^{(2)}), \cdots, F_m^{(j)}(x_m^{(j)}))$.

By stages listed above, we have arbitrarily obtained j parts from n parts, and $n-j$ remains of n parts naturally become a cluster, and denote $X_{(1)}^m, X_{(2)}^m, \cdots, X_{(n-j)}^m$ as their lives respectively. $X_1, X_2, \cdots, X_n$ are divided into $X_m^{(1)}, X_m^{(2)}, \cdots, X_m^{(j)}$ and $X_{(1)}^m, X_{(2)}^m, \cdots, X_{(n-j)}^m$ in the mth combination. Denote $F_{(1)}^m(x_{(1)}^m)$, $F_{(2)}^m(x_{(2)}^m), \cdots, F_{(n-j)}^m(x_{(n-j)}^m)$ as the distribution of $X_{(1)}^m, X_{(2)}^m, \cdots, X_{(n-j)}^m$. Then from Eq. (2)the copula of $X_{(1)}^m, X_{(2)}^m, \cdots, X_{(n-j)}^m$ is given by

$$H_{(n-j)}^m(x_{(1)}^m, x_{(2)}^m, \cdots, x_{(n-j)}^m) = C_{(n-j)}^m(F_{(1)}^m(x_{(1)}^m), \cdots, F_{(n-j)}^m(x_{(n-j)}^m)).$$

Let $\widehat{X}^m_{(n-j)}=\max(X^m_{(1)},X^m_{(2)},\cdots,X^m_{(n-j)})$, $\check{X}^{(j)}_m=\min(X^{(1)}_m,X^{(2)}_m,\cdots,X^{(j)}_m)$

$$\begin{aligned}\widehat{F}^m_{(n-j)}(t)&=P\{\widehat{X}^m_{(n-j)}\leqslant t\}\\&=P\{\max(X^m_{(1)},X^m_{(2)},\cdots,X^m_{(n-j)})\leqslant t\}\\&=C^m_{(n-j)}(F^m_{(1)}(t),\cdots,F^m_{(n-j)}(t))\end{aligned}\tag{3}$$

$$\begin{aligned}\check{F}^{(j)}_m(t)&=P\{\check{X}^{(j)}_m\leqslant t\}\\&=1-P\{\check{X}>t\}\\&=1-P\{\min(X^{(1)}_m,X^{(2)}_m,\cdots,X^{(j)}_m)>t\}\\&=1-\sum\operatorname{sgn}(c)C^{(j)}_m(c)\end{aligned}\tag{4}$$

where $c=(F^{(1)}_m(t),F^{(2)}_m(t),\cdots,F^{(j)}_m(t))$, $t=t$ 或∞, $F^{(i)}_m(\infty)=1$,

$$\operatorname{sgn}(c)=\begin{cases}1, & if\ F^{(i)}_m(t)\neq 1\ for\ an\ even\ number\ of\ i's\\-1, & if\ F^{(i)}_m(t)\neq 1\ for\ an\ odd\ number\ of\ i's\end{cases}\qquad i=1,2,\cdots,j$$

The joint distribution of $\widehat{X}^m_{(n-j)}$ and $\check{X}^{(j)}_m$ is obtained by

$$\widetilde{H}^{(j)}_m(\hat{x},\check{x})=P\{\widehat{X}^m_{(n-j)}\leqslant\hat{x},\check{X}^{(j)}_m\leqslant\check{x}\}$$

By Sklar's theorem, there exists a bivariate copula $\widetilde{C}^{(j)}_m$, such that

$$\widetilde{H}^{(j)}_m(\hat{x},\check{x})=\widetilde{C}^{(j)}_m(\widehat{F}^m_{(n-j)}(\hat{x}),\check{F}^{(j)}_m(\hat{x}))$$

By Eq. (3) and Eq. (4), we have the following

$$\widetilde{H}^{(j)}_m(\hat{x},\check{x})=\widetilde{C}^{(j)}_m(C^m_{(n-j)}(F^m_{(1)}(\hat{x}),F^m_{(2)}(\hat{x}),\cdots,F^m_{(n-j)}(\hat{x})),1-\sum\operatorname{sgn}(c)C^{(j)}_m(c)).$$

Where $c=(F^{(1)}_m(\check{x},F^{(2)}_m(\check{x}),\cdots,F^{(j)}_m(\check{x}))$, $\check{x}=\check{x}$ or ∞, $F^{(i)}_m(\infty)=1$,

$$\operatorname{sgn}(c)=\begin{cases}1, & if\ F^{(i)}_m(\check{x})\neq 1\ for\ an\ even\ number\ of\ i's\\-1, & if\ F^{(i)}_m(\check{x})\neq 1\ for\ an\ odd\ number\ of\ i's\end{cases}\qquad i=1,2,\cdots,j$$

$$\begin{aligned}R^{(j)}_m(t)&=P\{\max(X^m_{(1)},\cdots,X^m_{(n-j)})\leqslant t,\min(X^{(1)}_m,\cdots,X^{(j)}_m)>t\}\\&=P\{\max(X^m_{(1)},\cdots,X^m_{(n-j)})\leqslant t\}-P\{\max(x^m_{(1)},\cdots,X^m_{(n-j)})\leqslant t,\min(X^{(1)}_m,\cdots,X^{(j)}_m)\leqslant t\}\\&=P\{\widehat{X}^{n-j}\leqslant t\}-P\{\widehat{X}^m_{(n-j)}\leqslant t,\check{X}^{(j)}_m\leqslant t\}\\&=P\{\widehat{X}^{n-j}\leqslant t\}-\widetilde{C}^{(j)}_m(\widehat{F}^m_{(n-j)}(t),\check{F}^{(j)}_m(t))\\&=C^m_{(n-j)}(F^m_{(1)}(t),\cdots,F^m_{(n-j)}(t))-\widetilde{C}^{(j)}_m(C^m_{(n-j)}(F^m_{(1)}(t),F^m_{(2)}(t),\cdots,F^m_{(n-j)}(t),1-\sum\operatorname{sgn}(c)C^{(j)}_m(c))\end{aligned}$$

By the Frechet-Hoeffding bounds, we can obtain

$$\max(C^m_{(n-j)}-\sum\operatorname{sgn}(c)C^{(j)}_m(c),0)\leqslant\widetilde{C}^{(j)}_m\leqslant\min(C^m_{(n-j)},1-\sum\operatorname{sgn}(c)C^{(j)}_m(c))$$

$$\max(C^m_{(n-j)}-\sum\operatorname{sgn}(c)C^{(j)}_m(c)-1,0)\leqslant R^{(j)}_m(t)\leqslant\min(C^m_{(n-j)},\sum\operatorname{sgn}(c)C^{(j)}_m(c))$$

We can show that $\sum\operatorname{sgn}(c)C^{(j)}_m(c)$ increases as the $C^{(j)}_m$ increases. When the Copula C of all parts increases, $C^{(j)}_m$ and $C^m_{(n-j)}$ increase, then the lower and upper bound of $R^{(j)}_m(t)$ increase too.

The reliability of system is given by

$$\begin{aligned}R(t)&=\sum_{j=k}^{n}\left\{\sum_{m=1}^{C^j_n}P\{X^m_{(1)},\cdots,X^m_{(n-j)}\leqslant t<X^{(1)}_m,\cdots,X^{(j)}_m\}\right\}\\&=\sum_{j=k}^{n}\left\{\sum_{m=1}^{C^j_n}P\{\max(X^m_{(1)},\cdots,X^m_{(n-j)})\leqslant t,\min(X^{(1)}_m,\cdots,X^{(j)}_m)>t\}\right\}\\&=\sum_{j=k}^{n}\left\{\sum_{m=1}^{C^j_n}\left[C^m_{(n-j)}(F^m_{(1)}(t),\cdots,F^m_{(n-j)}(t))-\widetilde{C}^j_j(C^m_{(n-j)}(F^m_{(1)}(t),\cdots,F^m_{(n-j)}(t),1-\sum\operatorname{sgn}(c)C^{(j)}_m(c)))\right]\right\}\end{aligned}\tag{5}$$

where $c=(F^{(1)}_m(t),F^{(2)}_m(t),\cdots,F^{(j)}_m(t))$, $t=t\ or\ \infty$, $F^{(i)}_m(\infty)=1$.

$$\operatorname{sgn}(c)=\begin{cases}1, \ if\ F_m^{(i)}(t)\neq 1 \ for\ an\ even\ number\ of\ i's \\ -1, \ if\ F_m^{(i)}(t)\neq 1\ for\ an\ odd\ number\ of\ i's\end{cases} \qquad i=1,2,\cdots,j$$

Then $R(t)=\sum_{j=k}^{n}\left\{\sum_{m=1}^{C_n^m} R_m^{(j)}(t)\right\}$.

From above, we can conclude that $C_m^{(j)}$ and $C_{(n-j)}^m$ increase as Copula C increases, then the reliability of system increases. Therefore, we want to improve the reliability of vote system by increasing the Copula$C$ of parts.

2 Conclusion

The methodology developed in this paper resorts to copula functions for measuring the reliability of vote system consisting of dependence parts in the reliability system. We use this methodology to investigate the reliability measure of vote system consisting of dependence parts and the change relation with the copula function. To investigate the reliability measure of vote system, the hypothesis of dependency parts is the extension of independence and linear correlative parts.

References:

[1] Pakinson D B. Reliability bounds for dependent failures [J]. IEEE Tvansaction on Reliability, 1988, 17(1): 54－56.

[2] Feng Y S. A method for computing structure system reliability with high accuracy [J]. Journal of Computers and Structures, 1989, 33(1): 1－5.

[3] 曹晋华，程　侃. 可靠性数学引论 [M]. 北京:科学出版社,1986: 1－86.

[4] Roger B. Nelsen. An introduction to Copula [M]. New York: Springer-Verlag, 1999, 18－42.

[5] 易文德. 简单系统相依部件的可靠性 [J]. 渝西学院学报,2005, 2: 5－8.

[6] 易文德，赵联文. 相依部件混合系统的可靠性 [J]. 重庆文理学院学报,2006, 10: 4－7.

基于 Copula 函数的相依部件表决系统的可靠性研究

易文德[1]，　卫贵武[2]

1. 重庆文理学院 数学与计算机科学系,重庆 永川 402160;　2. 重庆文理学院 经济与管理系,重庆 402160

摘要: 在可靠性理论中,部件相互独立是相依的特殊情况.可靠性数学研究系统的可靠性时,都是在假设部件之间相互独立的条件下进行的.本文应用 Copula 函数研究部件相依时表决系统的可靠性,分析系统的可靠度与 Copula 函数间的关系,揭示部件相依关系对表决系统可靠度的影响规律.

关键词: 可靠性;表决系统;Copula 函数;相依;独立

责任编辑　张　枸