文章编号: 1000-5471(2007)06-0048-04

Sojourn Time of Exhaustive Service Polling System with Bulk Arrival and Bernoulli Feedback①

LEI Yu-jie[1], LU Wei-chang[2]

1. Department of Mathematics, The Third Military Medical University, Chongqing 400038, China;
2. Chongqing Communication Institute, Chongqing 400035, China

Abstract: This paper studies exhaustive service polling system with bulk arrival and Bernoulli feedback, gives the distributions of sojourn times, and some other results in stationary condition.

Key words: polling system; bulk arrival; bernoulli feedback; exhaustive service; sojourn time

CLC number: O226 **Document code**: A

Since the early 1970's, extensive study was carried out on a polling system[1-3]. Having studied asymmetric exhaustive service polling system with bulk arrival and Bernoulli feedback. In former researches[4-6], we got the queue length distribution in stationary condition. In this paper, we want to obtain the distribution of sojourn time and some other results.

A polling system is a system of multiple queues accessed in cyclic order by a single server. We refer to a user requesting service as a station, and the time needed to switch service from one station to another as walking time. The message arrival processes at station $i\{N_i(t), t \geqslant 0\}$ are assumed to be Poisson with rate λ_i. There is the ξ_i message arrival at station i each time, which is a random variable of positive integer, $E(\xi_i) = 1/\alpha_i$. There is an infinite buffer capacity to store waiting messages at each station. The exhaustive service means the server will continue to serve a station until its buffer is emptied. Let B_i be service time of a message at station i, and $B_i(t)$ be the distribution function of B, $E(B_i) = 1/\beta_i$. When a message at station i completes its service, it departs from the system with probability σ_i and is fed back to the end of the queue with probability $1-\sigma_i$. Let

N = number of station in the system,

R_i = walking time from station i to station $i+1$,

$R_i^*(s) = E(e^{-sR_i})$, $r_i = E(R_i)$, $r = \sum_{i=1}^{N} r_i$, $\rho = \sum_{i=1}^{N} \rho_i$,

$L_i(t)$ = numbers of messages at station i at time t,

$\tau_i(m)$ = polling instant of station i in the mth polling cycle,

① 收稿日期: 2007-04-28
作者简介: 雷玉洁(1970-), 女, 重庆璧山人, 副教授, 主要从事排队论及数理统计研究.

$G_i(Z_1, Z_2, \cdots, Z_N) = E[\prod_{j=1}^{N} Z_j^{L_j(\tau_i)}]$, $\widetilde{G}_i(Z) = E(Z_j^{L_i(\tau_i)})$,

Q_i = numbers of messages at station i when a message completes its service at station i, $\widetilde{Q}_i(Z) = E(Z^{Q_i})$,

C_i = cycle time polling station i, i. e. the time interval from $\tau_i(m)$ to $\tau_i(m+1)$,

$C_i^*(s) = E(e^{-sC_i})$, I = intervisit time visiting station i, $I_i^*(s) = E(e^{-sI_i})$,

δ_i = elapsed time (age) of I_i, r_i = remaining time of I_i,

S_{i1} = sojourn time of a message of station i for the first time,

S_i = total sojourn time of a message of station i,

$S_{i1}^*(s) = E(e^{-sS_{i1}})$, $S_i^*(s) = E(e^{-sS_i})$, $i, j = 1, 2, \cdots, N$.

In this paper, we assume that all arrival processes, all service times, all walking times and all arrival bulk sizes mutually are independent, and ρ is less than 1.

In the following researches, we let $B_i \sim \Gamma(1, \beta_i)$, $\xi_i \sim Geo(\alpha_i)$, $i = 1, 2, \cdots, N$

Lemma 1　Let $M_i(t)$ be the number of message feedback at station i in interval $(o, t]$ when the server is at station i each time. Then $\{M_i(t), t \geqslant 0\}$ is Poisson process with rate $\bar{\sigma}_i\beta_i$.

The proof of Lemma 1 is simple.

Lemma 2　Let $P_i(S_1, S_2)$ be LST of (δ_i, r_i), then

$$P_i(S_1, S_2) = \frac{(1-\rho)[I_i^*(S_1) - I_i^*(S_2)]}{r(1-\rho_i)(S_1 - S_2)} = \frac{(1-\rho)\left[\widetilde{G}_i\left(\frac{S_1 - \lambda_i}{\bar{\alpha}_i S_1 - \lambda_i}\right) - \widetilde{G}_i\left(\frac{S_2 - \lambda_i}{\bar{\alpha}_i S_2 - \lambda_i}\right)\right]}{r(1-\rho_i)(S_1 - S_2)} \quad (1)$$

$$\delta_i^*(s) = r_i^*(s) = \frac{(1-\rho)\left[1 - \widetilde{G}_i\left(\frac{S - \lambda_i}{\bar{\alpha}_i S_2 - \lambda_i}\right)\right]}{r(1-\rho_i)s} \quad (2)$$

$$E(\delta_i) = E(V_i) = \frac{(1-\rho)E(I_i^2)}{2r(1-\rho_i)} \quad (3)$$

Proof see [4].

Lemma 3

$$S_{i1}^*(s) = \frac{\rho_i\sigma_i(1-\rho)B_i^*(\lambda_i s/g_i)[(1-\sigma_i\alpha_i)s - g_i]}{\lambda_i r\{s - g_i - B_i^*(\lambda_i s/g_i)[(1-\sigma_i\alpha_i)s - g_i]\}} + \frac{\beta_i(1-\rho)(\alpha_i\beta_i + s)\left[\widetilde{G}_i\left(\frac{\beta_i}{\beta_i + s}\right) - \widetilde{G}_i\left(\frac{s - \lambda_i}{\bar{\alpha}_i s - \lambda_i}\right)\right]}{r(\beta_i + s)(\alpha_i\beta_i - \lambda_i + s)} \quad (4)$$

where $g_i = \lambda_i + \bar{\sigma}_i\beta_i$

Proof (ⅰ) When the message arrives at station i, if the server is being at station i, we have

$$Q_i = \sum_{k=1}^{y_y(S_{i1})} \xi_{ik} \quad (5)$$

From which, be Lemma 1, we have

$$\widehat{Q}_i(Z) = S_{i1}^*[g_i - g_i\xi_i(Z)] = S_{i1}^*\left(\frac{g_i - g_i z}{1 - \bar{\alpha}_i z}\right)$$

By theorem 2[4], we have

$$S_{i1}^*(s) = \widetilde{G}_i\left[\frac{s - g_i}{\bar{\alpha}_i s - g_i}\right] = \frac{(1-\rho)\sigma_i B_i^*(\lambda_i s/g_i)[(1-\sigma_i\alpha_i)s - g_i]}{\lambda_i r\{s - g_i - B_i^*(\lambda_i s/g_i)[(1-\sigma_i\alpha_i)s - g_i]\}} \quad (6)$$

(ⅱ) When the message arrive at station i, if the server is being at station i, then we have

$$S_{i1}=r_i+\sum_{k=1}^{\sum_{l=1}^{N_i(\delta_i)}\xi_{il}}B_{ik}+B_i \tag{7}$$

Where B_{i1}, B_{i2}, B_{i3}, $\cdots$ are $i \cdot i \cdot d \cdot$ random variables.

By theorem 3 and theorem 2[4], we have

$$S_{i1}^*(s)=E(e^{-sS_{i1}})=B_i^*(s)E\left[\exp\left\{-s\left(r_i+\sum_{k=1}^{\sum_{l=1}^{N_i(\delta_i)}\xi_{il}}B_{ik}\right)\right\}\right]$$

$$=B_i^*(s)\int_0^\infty\int_0^\infty e^{-sx}E\left[\exp\left\{-s\sum_{k=1}^{x}B_{ik}\right\}\right]dP\{v_i\leqslant x,\ \delta_i\leqslant y\}$$

where $x=\sum_{l=1}^{N_i(y)}\xi_{iL}$. Because

$$E\left[\exp\left\{-s\sum_{k=1}^{x}B_{ik}\right\}\right]=\tilde{x}[B_i^*(s)]\text{and }\tilde{x}(Z)=E(Z^*)=e^{-[\lambda_i-\lambda_i\tilde{\xi}(Z)]y}.$$

Therefore

$$E\left[\exp\left\{-s\sum_{k=1}^{x}B_{ik}\right\}\right]=e^{-[\lambda_i-\lambda_i\tilde{\xi}(B_i^*(s))]y}$$

From which, we have

$$S_{i1}^*(s)=B_i^*(s)P(\lambda_i[1-\tilde{\xi}_i(B_i^*(s))],\ s)=\frac{\beta_i}{\beta_i+s}P\left(\frac{\lambda_i s}{\alpha_i\beta_i+s},\ s\right)$$

$$=\frac{(1-\rho)\beta_i(\alpha_i\beta_i+s)\left[\tilde{G}_i\left(\frac{\beta_i}{\beta_i+s}\right)-\tilde{G}_i\left(\frac{s-\lambda_i}{\bar{\alpha}_i s-\lambda_i}\right)\right]}{r(1-\rho_i)(\beta_i+s)(\alpha_i\beta_i-\lambda_i+s)s} \tag{8}$$

Furthermore, at arbitrary time, the probability of the server to be at station i is given by

$$E(T_i)/E(C_i)=\frac{r\rho_i}{1-\rho}/\frac{r}{1-\rho}=\rho_i \tag{9}$$

Therefore, From (6), (7) and (8), in general case we have (4). Thus, the proof of Lemma 3 is completed.

Theorem 4

$$S_i^*(s)=\frac{\sigma_i S_{il}^*(s)}{1-\bar{\sigma}_i\tilde{Q}_i(f(s))} \tag{10}$$

Where $S_{i1}^*(s)$ is given by (4), $\tilde{Q}_i(z)=\dfrac{(1-\rho)\sigma_i B_i^*(\lambda_i-\lambda_i\tilde{\xi}_i(Z))(\sigma_i+\bar{\sigma}_i Z)}{\lambda_i r\{Z-(\sigma_i+\bar{\sigma}_i Z)B_i^*(\lambda_i-\lambda_i\tilde{\xi}_i(Z))\}}$ is given by [4],

$$f(s)=\frac{\sqrt{5\bar{\sigma}_i^2\beta_i^2+4\lambda_i^2\bar{\alpha}_j^2+4\lambda_i\beta_i\bar{\sigma}_i(1+\bar{\alpha}_j)-4\bar{\sigma}_i\beta_i s}-(\bar{\sigma}_i\beta_i+2\lambda_i\bar{\alpha}_i)}{2\bar{\sigma}_j\beta_i}$$

Proof. Since we have

$$S_i=S_{i1}+S_{i2}+\cdots S_{i\eta_i},\ i=1,\ 2,\ \cdots,\ N \tag{11}$$

Where $\eta_i\sim Geo(\sigma_i)$. S_{i1}, S_{i2}, S_{i3}, $\cdots$ are mutually independent random variables, and S_{i2}, S_{i3}, S_{i4}, $\cdots$ have identical distribution. Note that $Q_i=\sum_{k=1}^{N_i(s_{i2})}\xi_{ik}+M_i(S_{i2})$, we get

$$\tilde{Q}_i(Z)=S_{i2}^*\left(\bar{\sigma}_i\beta_i+\lambda_i-\bar{\sigma}_i\beta_i Z-\frac{\lambda_i\alpha_i Z}{1-\bar{\alpha}_i Z}\right) \tag{12}$$

From (11) and (12), we can get (10).

References：

［1］ Takagi H. Analysis of Polling System［M］. Massachusetts：The MIT Press，1986.

［2］ DEMORES L F M. Comments on 'Analysis and applications of a multiqueue Cyclic Service Systems with Feedback'［J］. IEEE Trans Commun，1990，40(1)：148－149.

［3］ Sun Rong-heng. Asymmetric Exhaustive Service Polling System with Bernoulli Feedback［J］. 重庆大学学报自然科学版，2001，24(2)：127－131.

［4］ LEI Yu-jie，LU Wei-chang，Analysis of Exhaustive Service Polling System with Bulk Arrival and Bernoulli Feedback［J］. 西南师范大学学报自然科学版，2006，31(5)：57－61.

［5］ 雷玉洁. 具有贝努利反馈的批到达门限服务与 1 -有限服务轮询系统［J］. 西南师范大学学报自然科学版，2002，27(3)：293－297.

［6］ 雷玉洁，孙荣恒. 批到达穷尽服务轮询系统分析［J］. 重庆大学学报自然科学版，1999，22(6)：121－126.

具有批到达和贝努里反馈的穷尽服务轮询系统的逗留时间

雷玉洁[1]，　鲁韦昌[2]

1. 重庆第三军医大学 数学教研室，重庆 400038；2. 重庆通信学院 基础部，重庆 400035

摘要：研究了具有批到达和贝努里反馈的穷尽服务轮询系统，给出了在平稳状态下逗留时间的分布以及其他一些结果.

关键词：轮询系统；批到达；贝努里反馈；穷尽服务；逗留时间

责任编辑　张　枸