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Stationary Black Hole is the System within the Region of Total Unbounded Energy Spectrum①

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Abstract: The basic feature of energy spectrum of stationary black hole has been studied. It is shown that the stationary black hole is a system within the region of total unbounded energy spectrum; outside of the external-horizon the energy gap between the positive and negative energy spectrum is disappearance in the surface of external-horizon, and outside of the external-horizon, the black-body radiation system or the positive energy spectrum system can be easy excited. Otherwise, the region within the external-horizon of black hole, according to Landau's negative energy spectrum theory, the negative energy spectrum system can be easy excited, therefore, the external-horizon of black-hole is the juncture surface of positive and negative energy spectrum systems, outside of the external-horizon of black hole the black hole emerge a positive energy spectrum system, and within the range of the external-horizon of black hole the black hole emerges a negative energy spectrum system. In view of the above, a criterion is put forward, and making use of this criterion, it can easily determine the type of thermodynamics of the black hole.

Key words: black hole; external-horizon; total unbounded energy spectrum; criterion

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Here says of criterion is a standard according which we can be to point some system(for example the black hole) to belong certain type of thermodynamics, that is a positive energy spectrum(PES) system or a negative energy spectrum(NES) system. Obviously, this criterion is not necessary for that is given PES or NES system, but for all unbounded energy spectrum, i.e. total unbounded energy spectrum(TUES), the criterion is quite necessary. Because, if a system(for example, the black hole) within the region of TUES, the system is both PES system and NES system. There has arisen a problem, that the system is in what case and the system is belong to what type of thermodynamics, the PES or NES thermodynamics is?

1 The Characteristics of Energy Spectrum of Black Hole

According as Landau's negative energy spectrum theory[1], if we judge of the energy spectrum of black hole is PES or NES, we must study the structure of energy spectrum of black hole. No influence in generally, proceed from simplification, we study the characteristics of structure in the energy spectrum of the static-spherical symmetry black hole, the Schwarzchild(SW) black hole alone. We know that the square of 4-momentum ($p_u p^u$) in 4-flat space-time is an invariant by Lorentz transformations, that is[2]:

$$p_u p^u + m^2 = -\varepsilon + p^2 + m^2 = 0 \tag{1}$$

thus we have

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$$\varepsilon_{\pm}=\pm\sqrt{p^2+m^2},\qquad \varepsilon_{\pm}^{0}=\pm m \tag{2}$$

where p is the usual 3-dimensional momentum, m is the mass of particle, Eq. (2) shows that, in 4-flat time-space, the free particle systems have equally two energy spectrum, one is a PES, in which the bounded below-norm is $\varepsilon_{+}^{0}=+m$, the energy spectrum $\{\varepsilon_{+}\}$ in the region is: $\varepsilon_{+}^{0}\leqslant\varepsilon_{+}<\infty$; the other is a NES, in which the bounded above-norm is $\varepsilon_{-}^{0}=-m$, the energy spectrum $\{\varepsilon_{-}\}$ in the region is: $\varepsilon_{-}^{0}\geqslant\varepsilon_{-}>-\infty$. Between the PES and NES, there has a forbidden band gap: $\Delta\varepsilon_{+}^{0}=\varepsilon_{+}^{0}-\varepsilon_{-}^{0}=2m$. From above, the time-space is a flat-time-space, in which the gravitational field is non-existence. And now lead the gravitational field into the flat time-space, force the time-space warping. For simple, we assume that the warpage of time-space is static spherical symmetry (i. e. the SW-time-space). By using the general relativity and relativistic quantum theory, the literature [3] had been led to the rigorous expressions of $\varepsilon_{\pm}$ and $\Delta\varepsilon_{\pm}$ as follows

$$\varepsilon_{\pm}=\pm\frac{\sqrt{x^2-1}}{k}\left\{\frac{CP_y^2}{D}(1-y^2)(x^2-y^2)^8+\frac{C^2P_\phi^2}{D^4(1-y^2)}+\frac{C}{D}(mk)^2\right\}^{1/2} \tag{3}$$

$$\Delta\varepsilon_{\pm}=\varepsilon_{+}-\varepsilon_{-}=2\frac{\sqrt{x^2-1}}{k}\left\{\frac{CP_y^2}{D}(1-y^2)(x^2-y^2)^8+\frac{C^2P_\phi^2}{D^4(1-y^2)}+\frac{C}{D}(mk)^2\right\}^{1/2} \tag{4}$$

where k is a positive read number; p_y and p_φ are the components of momentum in y and ϕ respectively; C, D are decided by the equations

$$C=(x^2-y^2)^8\qquad D=[(x+1)(x^2-y^2)^4+(3x^2+1)]^2$$

here x and y are the dimensionless space coordinates, especially, the x is very important, in fact that, if $x=1$, it shows the external-horizon of black hole, and $\lim x\to\infty$, it shows that the observers are far away from black hole i. e. observer tends to flat time-space.

From Eq(3)and(4),we obtain two limit equations as below

$$\left.\begin{aligned}&\lim_{x\to1}\varepsilon_{\pm}=0,\qquad &&\lim_{x\to1}\Delta\varepsilon_{\pm}=0,\\&\lim_{x\to\infty}\varepsilon_{\pm}=\pm m,\qquad &&\lim_{x\to\infty}\Delta\varepsilon_{\pm}=2m.\end{aligned}\right\} \tag{5}$$

It follows that:

(1)The SW black hole is a system within the range of TUES.

Eq. (5) shows that the forbidden band-gap between PES and NES disappears as the observer tends to external-horizon of black hole. It is shown that outside the external-horizon of black hole the PES and NES are jointed together in external-horizon of black hole and form a TUES as follows

$$-\infty<\varepsilon_a<\infty,\qquad \varepsilon_a\in\bigcup_{i=\pm}\varepsilon_i \tag{6}$$

Obviously, the NES arises, owing to the symmetry of flate time-space, i. e. the squar of 4-momentum $p_u p^u$ is an invariant in Lorenz transformation, but is not the existence of the gravitational field. However, the existence of gravitational field is really the basic reason leads to disappearance of the forbidden band gap in external-horizon of black hole, and join the PES and NES together to form a TUES as Eq. (6). Since the gravitational field within the black hole is stronger than that outside the black hole, and so it cannot be to produce the forbidden band gap within the black hole once again, the energy spectrum inside the black hole must be still a TUES (Fig. 1).

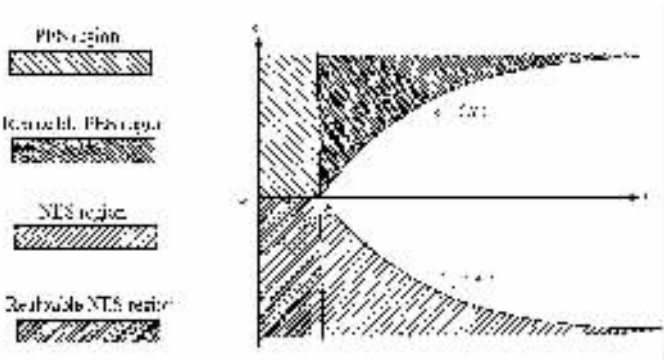


Fig. 1 The Structure of Energy Spectrum of SW Black Hole

(2)Inside of the black hole the system is easily formed of NES system; Outside of the black hole the

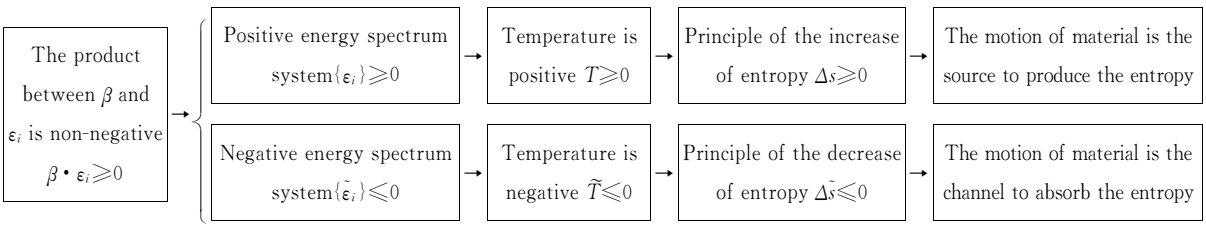
system is easily formed of PES system .

According as Landau's NES theory, by ues of the TOV equation of the internal field of SW black hole, the structure equation of internal field of SW black hole, we had rigorously demonstrated that the internal gravitational field of SW black hole is satisfied with the condition formed of the NES system. Therefore, inside the external-horizon of SW black hole is the region in which the system is easily formed of the NES system. But outside the external-horizon of SW black hole is the region in which the system is easily to form the black body radiation system, that is to say, outside the black hole is easily to form the PES system.

The above, even if the characteristics of energy spectrum are aimed at the SW black hole, which are also suitable for other stationary block hole. It is because that the difference between the SW black hole and others stationary black hole are the only finite motions for others stationary black hole (for example, in the other stationary black hole has rotation and electromagnetic field except the stronger gravitational field), these finite motion of other stationary black hole is only to change the distribution of levels in TUES of stationary black hole but cannot change the basic structure in TUES of stationary black hole. Therefore, the stationary black hole is also a system within the region of TUES. And hence, the stationary black hole is a PES system in some case, or else is a NES system in others case. That is to say the stationary black hole can be both PES system and NES system, here a question has arison, that the stationary black hole is what case and what system is (i. e. which is PES system or is NES system). In view of above, if we convenient to decide the type of thermodynamics of black hole, the criterion of type of thermodynamics must be established. The following is the basic main points for the establishment of the criterion of type of thermodynamics.

2 The Logical Procedure for the Theory of Thermedynamics

The thermodynamics has a theoretical logical procedure as follows



There have two laws which are the crux to determine the logical procedure of thermodynamics, and now, we will present this two laws in brief and clear.

a) The law of the product $\beta\cdot\varepsilon_i$ is non-negative[4].

In reference-document [4] the law of non-negative product $\beta\cdot\varepsilon_i$ had been exactly proven, that is

$$\beta\cdot\varepsilon_i\geqslant 0 \tag{7}$$

If Eq. (7) were not hold, not only the system can't be stationary, but also there will emerge many unphysical results, such as negative occupation number of particles, negative probability and hollow-Fermi ball, ete , hence, it follows that, for PES system the temperature must be positive: $T\geqslant 0$, owing to the energy energy spectrum is positive: $\{\varepsilon_i\}\geqslant 0$; for NES system the temperature must be negative $\tilde{T}\leqslant 0$, owing to the energy spectrum is negative: $\{\tilde{\varepsilon}_i\}\leqslant 0$. Since the temperature T(or $\tilde{T}$) is introduced before the entropy S and so the temperature T(or $\tilde{T}$) is called the basic-temperature.

b) The evolution law of entropy in PES and NES system[5]

The Clausius principle of increase of entropy is not universal law which is only suitable for the evolu-

tion of entropy in PES systems. In decument [5] we had rigorously proven that the spontaneous evolution law of entropy is determined by the symbol of basic temperature, that is if the temperature is positive the spontaneous evolution law of entropy must be increased, otherwise if the basic temperature is negative the spontaneous evolution law of entropy must be decreased. In fact that is impossible to design a "perpetual motion engine of the second kind", and hence, if the state of system change from state 1 into state 2, the heat $\Delta Q_a=(\sum_i \mathrm{d}Q_{ai})$ absorbed by the system in irreversible process have to be small than that heat $\Delta Q=(\sum_i \mathrm{d}Q)$ absorbed by the system in reversible process, that is

$$\Delta Q_a = \sum_i \mathrm{d}Q_i > \sum_i \mathrm{d}Q_{ai} = \Delta Q_a \tag{8}$$

thus the change of entropy from state 1 into state 2 must be

$$(\Delta S)_+ = \int_2^2 \mathrm{d}s = \int_1^2 \frac{\mathrm{d}Q}{T} > \int_1^2 \frac{\mathrm{d}Q_a}{T}, \qquad \text{if} \qquad T \geqslant 0 \tag{9}$$

$$(\Delta S)_+ = \int_2^2 \mathrm{d}s = \int_1^2 \frac{\mathrm{d}Q}{T} < \int_1^2 \frac{\mathrm{d}Q_a}{T}, \qquad \text{if} \qquad T \leqslant 0 \tag{9'}$$

where $(\Delta S)_+$ is the change of entropy of PES system and $(\Delta S)_-$ is the change of entropy of NES system. If the system is isolated or the process is adiabatic, it must be

$$(\Delta S)_+ > 0, \qquad T \geqslant 0^+ \tag{10}$$

$$(\Delta S)_- < 0, \qquad \widetilde{T} \leqslant 0^- \tag{10'}$$

Therefore, the entropy of the isolated system in PES system must be spontaneously increased in irreversible process, this is so called the Clausius principle of entropy increase, and the entropy of isolated system in NES system must be spontaneously decreased in irreversible process. Hence the motion of substance in PES isolated system is the source of production of entropy, and the motion of substance in NES isolated system is the ditch of absorbing of entropy. In NES system, the ability for absorbing positive entropy is measured by the reservation of positive entropy reserve, that is

$$\widetilde{S} = (\Delta S)_- = S_+ - S_i < 0, \qquad S_+ < S_i \tag{11}$$

where S_i is the stored positive entropy in initial state, and S_f is the surplus positive entropy in final state. while the surplus positive entropy S_f is equal to zero, i. e. $S_f=0$, the system reach the maximum of absorbing positive entropy. The above from this two laws we can immediately, establish the logical procedure for thermodynamic theory.

3 The Criterion of the Type of Thermodynamics

Since the black hole is a system within the TUES, thus any black hole may be PES system or NES system. But we actually want to know what is the condition the black hole is in what type of thermodynamics. In the case the establishment of criterion is very necessary and now according as the three fundamental theory, we express the criterion as follows:

While a process in which the material of black hole is transported from inside into outside of external-horizon of block hole and established a stationary or gradually stationary system. The system is necessarily a PES system, the basic temperature of it must be positive, $T\geqslant 0$, and the evolution of entropy of it must be according as the Clausius principle of entropy increase; while a process in which the material is absorbed by black hole from outside into inside of external-horizon of black hole, and established a stationary or gradually stationary system, the system is necessarily a NES system, the basic temperature of it must be negative, $\widetilde{T}\leqslant 0$, and the evolution of entropy of it must be according as the principle of entropy decrease.

There have been three theoretical grounds established the criterion of the type of thermodynamics

these grounds are below.

(1)From quantum field theory and general theory of relativity the basic feature of energy spectrum of stationary black hole had been determined, that is:

(a) The energy spectrum of stationary black hole is the TUES;

(b) The external-horizon of black hole is the junction surface of PES and NES.

(2) In accordance with the Landau's theory of NES, inside the stationary black hole the NES condition is exactly satisfied by the internal gravitational field. And so the region inside the external-horizon of black hole is easily to form the NES system. Other wise, according as Hawking's theory of quantum radiation, the region outside the external-horizon of black hole is easly to form the PES system.

(3)According as the logical procedure between the energy spectrum、temperature and the evolution law of entropy, the sign of temperature is determined by the symbol of energy spectrum, and the evolution law of entropy is determined by the symbol of temperature, that is to say the positive (negative) temperature is determined by the PES(NES); the evolution law of entropy increase (decrease) is established by the positive (negative) temperature. And so the criterion is established by this three theoretical grounds as above. Table 1 shows the contrast the inside and the outside of black hole for the characters and evolution laws of the system.

Table 1　The Contrast Between the Inside and Outside of Black Hole for the Characters and the Evolution Laws of the System

System	Energy spectrum	Basic temperature	The evolution law of entropy	Some otheres law
A system is formed by that matter is outside the external-horizon of black hole	Positive system, the energy spectrum $\{\varepsilon_i\}$ is $0\leqslant\varepsilon_i<\infty$	The tempemture of system is positive: $T\geqslant 0$	The evolution law of system is entropy increase. $dS\geqslant 0$	The distribation of number of particles is radiation law of black-body $\langle n\rangle=\left[\exp\left(\frac{\varepsilon_i}{k_B T}\right)\pm 1\right]^{-1}$
A system is formed by that matter is iuside the external-honzon of black hole	Negative system, the energy spectrum $\{\tilde{\varepsilon}_i\}$ is $-\infty<\{\tilde{\varepsilon}_i\}\leqslant 0$	The temperature of system is negative $\tilde{T}\leqslant 0$	The evolution law of system is entropy decrease. $d\tilde{S}\leqslant 0$	The distribution of number of particles is absorbing law of particle. $\langle\tilde{n}\rangle=\left[\exp\left(\frac{\tilde{\varepsilon}_i}{k_B\tilde{T}}\right)\pm 1\right]^{-1}$

If we view and analyze the characters and evolution laws of black hole in the light of criterion, we can immediately give a correct judgements.

4　Distinguish Between the True and Falseness of Evolution Law

4.1　True and Falseness of Bekenstein's Area-Entropy Law

Now, we study the true and falseness of Bekenstein's area-entropy law. The area-entropy law has answered a question that while the matter which is controlled by gravitational field is absorbed from outside into inside of black hole, how is the evolution of the entropy of black hole. Bekenstein stands for that the evolution of black hole is in accordance with the Clausius principle entropy increase, when he answered this problem. In other words, Bekenstein stands for that the black hole is a PES system, the horizon-temperature of black hole is un-negative: $T_+=\frac{\hbar k_+}{2\pi k_B C}\geqslant 0$[6], and thus the evolution law of entropy of black hole is the principle law of entropy increase, namely the entropy of black hole is proportional to the area A of horizon of black hole: $dS=\frac{k_B}{4}dA$. Table 2, according as Bekenstein's theory, the characters and evolution law of black hole is arranged in the table. If we compare Table 2 with the column 2 in Table 1, we find

that the Bekenstein's thermodynamics of black hole is absolutely wrong.

Table 2 The Expression of the Bekenstein's Shermodynamic of Black hole

System	Energy spectrum	Basic temperature	The evolution law of entropy	Some other law
A system is formed by that matter is inside the external-horizon of black hole	Positive system, the energy spectrum $\{\varepsilon_i\}$ is $0\leqslant\varepsilon_i<\infty$	The temperature of system is positive $T=\frac{\hbar k_+}{2\pi k_B C}\geqslant 0$	The evolution law of black hole is etropy increase: $dS=\frac{k_B}{4}dA\geqslant 0$	The distribution of number of particles is radiation law of black-body $\langle n\rangle=\left[\exp\left(\frac{\varepsilon_i}{k_B T}\right)\pm 1\right]^{-1}$

4.2 Hawking's Quantum Theory of Gravitational Radiation

From the theory of impossible elimination of vacuum fluctuation, Hawking put forward the quantum theory of gravitational radiation with regard to black hole, Hawking had rigorously proven the following results.

(i) With the region outside the external-horizon of black hole, the black hole must be producing radiation of black-body as below[7]

$$\langle n(\varepsilon)\rangle=\left[\exp\left(\frac{\varepsilon}{k_B T}\right)\pm 1\right]^{-1} \tag{12}$$

Eq. (12)shows that through the process of quantum of gravitational rediation, the matter of black hole must be evaporated from inside into outside the external-horizon of black hole.

(ii) The black-body radiation system is formed in the region outside of the external-horizon of black hole, the temperature of this radiation system Hawking had been rigorously proven un-negative, $T\geqslant 0$, and is given by[7]

$$T=\frac{\hbar k_+}{2\pi k_B c}\geqslant 0 \tag{13}$$

And then, we set the SW black hole as an example, by use of the theory of black-body radiation, we had proven that the radiation entropy of SW black hole is given by

$$S_R(M_R)=\frac{xa^3}{2M_B^2}\left(\frac{R}{1-R}\right),\qquad R=2\frac{M_R}{M_B}-\left(\frac{M_R}{M_B}\right)^2 \tag{14}$$

where M_R is the radiation mass, M_B is the initial mass of black hole, x and a are the function of $\hbar$, c, G and which are quite evidently large than 1, R is called the radiation function of SW black hole. We see that: if $M_R=0$, then the $R=0$, and the radiation entropy $S_R=0$; if $M_R=M_B$, then the $R=1$ and the radiation entropy $S_R(M_B)=\infty$, And hence, the radiation entropy S_R is rapidly increased function with M_R increase. Therefore, the Hawking radiation process is a entropy increased process. Table 3 is given that the energy spectrum、temperature and evolttion law of entropy of stationary black hole And now, compare the Table 3 with the column 1 in Table 1, we find that the law of Hawking quantum radiation is a quite correct law.

Table 3 Hawking's Quantum Theory of Gravitational Radiation

System	Energy spectrum	Basic temperature	The evolution law of entropy	Some other law
A system it formed by that matter is cutside the external horizon.	Positive energy spectrum $\{\varepsilon_i\}$ is $0\leqslant\varepsilon_{ji}<\infty$	Temperature is Positive $T=\frac{\hbar k_+}{2\pi k_B c}\geqslant 0$	The evlution law of black hole is the radiation entropy increase: $dS_R=\frac{xa^3\,dM_R}{(M_B-M_R)^3}\geqslant 0$	The distribution of number of radiation particles is radiation law of black body. $\langle n\rangle=\left[\exp\left(\frac{\varepsilon_i}{k_B T}\right)\pm 1\right]^{-1}$

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稳态黑洞是处于全无界能谱中的系统

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摘要： 对稳态黑洞能谱的基本特征进行了研究. 研究表明稳态黑洞是处于全无界能谱中的系统. 在其外视界之外正、负能谱之间禁带能隙消失在外视界面上. 在外视之外，黑体辐射系统或正能谱系统易于激发；反之，根据 Landau 的负能谱理论，在外视界之内负能谱系统易于激发. 因此，黑洞的外视界是正、负能谱系统的交接面，在黑洞外视界以外黑洞显示为正能谱系统，而在黑洞外视界以内黑洞又显示为负能谱系统. 有鉴于此，我们提出了一个判据，利用这个判据可以很容易地确定黑洞的热力学类型.

关键词： 黑洞；外视界；全无界能谱；判据

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