

# A New Class of Generalized Nonlinear Fuzzy Set-Valued Variational Inclusions Involving $(H, \eta)$ -Monotone Mappings<sup>①</sup>

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**Abstract:** In this paper, a new class of generalized nonlinear fuzzy set-valued variational inclusions in Hilbert spaces is introduced and studied, and an existence theorem of solutions and a new iterative algorithm for this class of variational inclusions is suggested and discussed. The results presented in this paper generalize and improve some recent results in this field.

**Key words:** variational inclusions;  $(H, \eta)$ -monotone mapping; fuzzy set-valued mapping; convergence; iterative algorithm

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Form 1989, [1-4] introduced and studied some existence theorems of solutions and some iterative algorithms for several kinds of fuzzy set-valued variational inclusions and inequalities. On the other hand, [5] studied a new class of generalized monotone mappings,  $(H, \eta)$ -monotone mappings.

Inspired and motivated by recent research works in this field, a new class of generalized nonlinear fuzzy set-valued variational inclusions in Hilbert spaces is introduced and studied. By using the resolvent operator associated with  $(H, \eta)$ -monotone mappings, an existence theorem of solutions and a new iterative algorithm for this class of variational inclusions is suggested and discussed. The results presented in this paper generalize and improve some recent results in [1-4,6].

## 1 Preliminaries

Let  $X$  be a real Hilbert space with a norm  $\|\cdot\|$  and an inner product  $\langle \cdot, \cdot \rangle$ , and  $\mathcal{F}(X)$  be a collection of all fuzzy sets over  $X$ . Let  $\hat{S}, \hat{T}, \hat{G}$  and  $\hat{P} : X \rightarrow \mathcal{F}(X)$  be four fuzzy mappings satisfying the condition

(\*) there exists four functions  $a, b, c, d : X \rightarrow [0, 1]$  such that for all  $x \in X$ , we have  $(\hat{S}_x)_{a(x)}, (\hat{T}_x)_{b(x)}, (\hat{G}_x)_{c(x)}, (\hat{P}_x)_{d(x)} \in CB(X)$ , where  $CB(X)$  denotes the family of all nonempty bounded closed subsets of  $X$ .

If letting  $S(x) = (\hat{S}_x)_{a(x)}, T(x) = (\hat{T}_x)_{b(x)}, G(x) = (\hat{G}_x)_{c(x)}, P(x) = (\hat{P}_x)_{d(x)}$ , then  $S, T, G$ , and

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