第 36 卷 第 3 期 西南师范大学学报(自然科学版) 2011 年 6 月
Vol. 36 No. 3 Journal of Southwest China Normal University (Natural Science Edition) Jun. 2011

文章编号:1000-5471(2011)03-0011-03

On Identities of Locally Left Seminormal Orthodox Cryptogroup[①]

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Abstract: The purpose of this paper is to give some identities of locally left quasinormal orthogroups and locally left seminormal orthogroups respectively. It also proved that a completely regular semigroup is a left seminormal cryptogroup if and only if it is a locally left seminormal orthodox cryptogroup.

Key words: variety; locally left seminormal orthodox cryptogroup; left seminormal cryptogroup

CLC number: O152.7 **Document code**: A

Throughout this paper, S stands for a completely regular semigroup and $E(S)$ is the set of idempotents of S, a^{-1} is the inverse of a in $\mathscr{H}$.

Recall that S is an orthogroup if $E(S)$ is a subsemigroup of S. An orthogroup S is a $\mathscr{C}$ orthogroup if $E(S)$ belongs to a class $\mathscr{C}$ of bands. If Green relation $\mathscr{H}$ is a congruence on S, then S is called a cryptogroup. A cryptogroup S is a $\mathscr{C}$ cryptogroup if $S/\mathscr{H}$ belongs to a class $\mathscr{C}$ of bands. A orthodox cryptogroup is an orthocryptogroup.

Denote by $\mathscr{L}(\mathscr{CR})$ all subvarieties of completely regular semigroups and let $\mathscr{V}$ be a subvariety of completely regular semigroups. S is a locally $\mathscr{V}$-semigroup if $eSe\in\mathscr{V}$ for any $e\in E(S)$. In fact, the locality method is very important in semigroups algebraic theory. We know that a completely regular semigroup S is a completely simple semigroup if and only if S is locally a group, and S is a normal cryptogroup if and only if S is locally a Clifford semigroup.

Now, we list some special notations, which we will use later.

$\mathscr{CR}$ is the class of completely regular semigroups; $\mathscr{O}$ is the class of orthogroups; S is called a orthogroup if $a^0b^0=(a^0b^0)^0$ for any $a,b\in S$; $\mathscr{BG}$ is the class of cryptogroups; S is a cryptogroup if $(ab)^0=(a^0b^0)^0$ for any $a,b\in S$; $\mathscr{OBG}$ is the class of orthocryptogroups; $\mathscr{LQNO}$ is the class of left quasinormal orthogroups; S is a left quasinormal orthogroup if $efg=efeg$ for any $e,f,g\in E(S)$; $\mathscr{LSNO}$ is the class of left seminormal orthogroups; S is called a left seminormal orthogroup if $efg=efgeg$ for any $e,f,g\in E(S)$.

All terminology and notations which are not explained can be found in references [1-3]. Now, we give the main results of the paper.

Lemma 1[2] $S\in L\mathscr{LRO}$ if and only if S satisfies the identity $axy=axy(ay)^0$.

Lemma 2[2] $\mathscr{LROBG}=\mathscr{LSNBG}$.

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① 收稿日期:2010-06-06
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Proposition 1 $S \in \mathscr{LQNO}$ if and only if S satisfies the identity $axb = axa^0b$.

proof Let $S \in \mathscr{LQNO}$. For all $e, f, g \in E(S)$, we have $efg = efeg$. Let $a \in S$, since $efg = efeg$, $aea^{-1} \in E(S)$ and Corollary II. 5. 2 in reference [1], we have

$$\begin{aligned} eaef &= ea(a^{-1}a)ef = ea(a^{-1}a)e(a^{-1}a)f = e(aea^{-1})af = e(aea^{-1})(aa^{-1})af = \\ & e(aea^{-1})e(aa^{-1})af = eaea^{-1}eaf = (ea)e(a^{-1}e)af = eaa^{-1}eaf = \\ & (eaa^{-1}eaa^{-1})af = (eaa^{-1})af = eaf \end{aligned}$$

So for any $x, b \in S$, we get $axb = aa^0xb^0 = a(a^0xa^0b^0)b = axa^0b$.

Conversely, if S satisfies the identity $axb = axa^0b$, one has $S \in \mathscr{LQNO}$.

Theorem 1 $S \in L\mathscr{LQNO}$ if and only if S satisfies the identity $axy = axy(ay)^0$.

proof Let $S \in L\mathscr{LQNO}$. For all $a, x, b, y \in S$, by Proposition II. 7. 3 in reference [1] and Proposition 1 we have

$$(yay)(yxy)(yby) = (yay)(yxy)(yay)^0(yby)$$

Substituting $a \to xa$, $x \to ax$ and $b \to y$, we get

$$(yxay)(yaxy)(yyy) = (yxay)(yaxy)(yxay)^0(yyy)$$

Postmultiplying it by $(y^3)^{-1}$, we have

$$(yxay)(yaxy)(y^3)^0 = (yxay)(yaxy)(yxay)^0(y^3)^0$$

Since $S \in \mathscr{CR}$, we have $y^0 = (y^3)^0$. So

$$(yxay)(yaxy) = (yxay)(yaxy)(yxay)^0$$

Premultiplying it by $(yaxyxay)^{-1}(yax)$, we get

$$(yaxyxay)^0(yaxy) = (yaxyxay)^0(yaxy)(yxay)^0$$

By Corollary II. 4. 3 in reference [1] and $(yaxyxay)^0 \mathscr{R} (yaxy)$, we have

$$yaxy = yaxy(yxay)^0$$

Premultiplying it by $(axy)^{-1}(ax)$, one gives $axy = axy(yxay)^0$. Postmultiplying it by $(ay)^0$, we obtain $axy = axy(ay)^0$.

Conversely, assume that $axy = axy(ay)^0$ for any $a, x, y \in S$. Let $e \in E(S)$, and $b, c, z \in eSe$. Then $(bc)^0 \leqslant e$, $(bcb)^0 \leqslant e$, $(bc)^0 \mathscr{R} (bcb)^0$, so

$$(bc)^0 = (bcb)^0(bc)^0e = (bcb)^0(bc)^0e((bcb)^0e)^0 = (bcb)^0$$

Postmultiplying it by $(b)^0$, we have $(bc)^0b^0 = (bc)^0$. Then $bc = bcb^0$, so $bcz = bcb^0z$. By Proposition 1 we get $S \in L\mathscr{LQNO}$. By Lemma 1 we get $L\mathscr{LQNO} = L\mathscr{LRO}$, by Proposition II. 8. 4 in reference [1] and Lemma 2 we have $L\mathscr{LQNOBG} = \mathscr{LSNBG}$.

Lemma 3 $L\mathscr{LQNO} \subseteq L\mathscr{LSNO}$.

proof Let $S \in \mathscr{LQNO}$. For any $e, f, g \in E(S)$, we have $efg = efeg$. Postmultiplying it by eg we get

$$efgeg = efegeg = efeg = efg$$

So, we have that $efg = efgeg$. Then we get $S \in \mathscr{LSNO}$, so $L\mathscr{LQNO} \subseteq L\mathscr{LSNO}$.

Proposition 2 $S \in \mathscr{LSNO}$ if and only if S satisfies the identity $axb = axb^0a^0b$.

proof Let $S \in \mathscr{LSNO}$. For any $e, f, g \in E(S)$ we have $efg = efgeg$. Then for any $a \in S$, we have $(afefa^{-1})(afefa^{-1}) = a(fefa^{-1}afefa^{-1}a)a^{-1} = afefa^{-1} \in E(S)$. Since $efg = efgeg$ and Corollary II. 5. 2 in reference [1], we have

$$\begin{aligned} eafef &= ea(a^{-1}a)fef = ea(a^{-1}a)(fe)f(a^{-1}a)f = \\ & e(afefa^{-1})af = e(afefa^{-1})aa^{-1}af = \\ & e(afefa^{-1})(aa^{-1})e(aa^{-1})af = \\ & (eaf)ef(a^{-1}eaf) = eafa^{-1}eaf = \\ & eaf(a^{-1}aa^{-1})e(aa^{-1}a)f = \\ & e(afa^{-1})(aa^{-1})e(aa^{-1})af = \end{aligned}$$

$$e(afa^{-1})(aa^{-1})af =$$
$$ea(a^{-1}afa^{-1}af) = ea\ (a^{-1}af) = eaf$$

So for any $x, b \in S$ we obetin $axb = aa^0xb^0b = a(a^0xb^0a^0b^0)b = axb^0a^0b$. So S satisfies the identity $axb = axb^0a^0b$.

Conversely, if S satisfies the identity $axb = axb^0a^0b$, one has $S \in \mathscr{LSNO}$.

Theorem 2　$S \in L\mathscr{LSNO}$ if and only if S satisfies the identity $axy = axy(ay)^0$.

proof　Let $S \in L\mathscr{LSNO}$. For all $a, x, b, y \in S$, by Proposition II. 7. 3 in reference [1] and Proposition 2 we have

$$(yay)(yxy)(yby) = (yay)(yxy)(yby)^0(yay)^0(yby)$$

Substituting $a \to xa$, $x \to ax$ and $b \to y$, we get

$$(yxay)(yaxy)(yyy) = (yxay)(yaxy)(yyy)^0(yxay)^0(yyy)$$

Postmultiplying it by $(y^3)^{-1}$, have

$$(yxay)(yaxy)(y^3)^0 = (yxay)(yaxy)(y^3)^0(yxay)^0(y^3)^0$$

Since $S \in \mathscr{CR}$, we have $y^0 = (y^3)^0$. So

$$(yxay)(yaxy) = (yxay)(yaxy)(yxay)^0$$

By the proof of Theorem 1 we have $axy = axy(ay)^0$.

Conversely, let S satisfies the identity $axy = axy(ay)^0$. By Theorem 1 and Lemma 3 we have $S \in L\mathscr{LSNO}$.

So we obtain:

Theorem 3　$L\mathscr{LRO} = L\mathscr{LQNO} = L\mathscr{LSNO} = [axy = axy(ay)^0]$.

Theorem 4　$L\mathscr{LROBG} = L\mathscr{LQNOBG} = L\mathscr{LSNOBG} = \mathscr{LSNBG}$.

Symmetrically, we get:

Theorem 5　$L\mathscr{RRO} = L\mathscr{RQNO} = L\mathscr{RSNO} = [axy = (ay)^0axy]$.

Theorem 6　$L\mathscr{RROBG} = L\mathscr{RQNOBG} = L\mathscr{RSNOBG} = \mathscr{RSNBG}$.

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关于局部左半正规纯正密码群并半群的等式

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摘要: 给出了关于局部左拟正规纯正密码群并半群和局部左半正规纯正密码群并半群的一些等式. 证明了完全正则半群是左半正规密码群并半群当且仅当它是局部左半正规纯正密码群并半群.

关键词: 簇；局部左半正规纯正密码群并半群；左半正规密码群并半群

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