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The Structure of Complete $\mathscr{J}^{*,\sim}$-Simple Semigroups①

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Abstract: A complete $\mathscr{J}^{*,\sim}$-simple semigroup is a generalized complete simple semigroup in the range of rpp semigroups. In this paper, a structure theorem for complete $\mathscr{J}^{*,\sim}$-simple semigroups in terms of normalized Rees matrix semigroups over some left cancellative monoids is provided.

Key words: r-ample semigroups; super-r-ample semigroups; complete $\mathscr{J}^{*,\sim}$-simple semigroups; normalized Rees matrix semigroups

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Among regular semigroups, the class of completely regular semigroups is the core subclass of the class of regular semigroups. It was noticed in Reference [1] that a completely regular semigroup can be expressed by a semilattice of completely simple semigroups. In addition, a recipe for manufacturing completely simple semigroups by using Rees matrix semigroups was provided in Reference [1].

Later on, completely regular semigroups were generalized to superabundant semigroups by means of $*$-Green's relations and a semilattice decomposition of superabundant semigroups was given in Reference [2]. Further, the matrix representation for completely $\mathscr{J}^*$-simple semigroups and the structure for superabundant semigroups were investigated in Reference [3].

Let S be a semigroup, $\mathscr{E}(S)$ the lattice of all equivalences on S. For any $\sigma\in\mathscr{E}(S)$, we call that $A\subseteq S$ a subset saturated by σ, if A is a union of some σ-classes of S. We call that S is σ-abundant, if every σ-class of S contains idempotents of S.

In order to further generalize completely regular semigroups in the range of rpp semigroups, new Green's relations, $(*,\sim)$-Green's relations on a semigroup were introduced in Reference [4]. The relations $\mathscr{L}^{*,\sim}$ and $\mathscr{R}^{*,\sim}$ are respectively defined as $\mathscr{L}^*$ and $\widetilde{\mathscr{R}}$. The intersection and the join of $\mathscr{L}^{*,\sim}$ and $\mathscr{R}^{*,\sim}$ are respectively denoted by $\mathscr{H}^{*,\sim}$ and $\mathscr{D}^{*,\sim}$. The relation $\mathscr{J}^{*,\sim}$ is defined by the rule that $a\,\mathscr{J}^{*,\sim}b$ if and only if $J^{*,\sim}(a)=J^{*,\sim}(b)$. Where, for any $a,b\in S$, $a\widetilde{\mathscr{R}}b$ if and only if for all $e\in E(S)$, $ea=a$ if and only

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if $eb=b$, and $J^{*,\sim}(a)$ is the smallest ideal containing a and saturated by $\mathscr{L}^{*,\sim}$ and $\mathscr{R}^{*,\sim}$.

According to Reference [4], a semigroup S is called an r-ample semigroup if S is $\mathscr{L}^{*,\sim}$-abundant and $\mathscr{R}^{*,\sim}$-abundant. An r-ample semigroup is called a super-r-ample semigroup, if S is $\mathscr{H}^{*,\sim}$-abundant. The class of super-r-ample semigroups forms a proper extension class of the class of superabundant semigroups.

It was shown in Reference [4] that $\mathscr{R}^{*,\sim}$ usually is not a left congruence on S even if S is an $\mathscr{R}^{*,\sim}$-abundant semigroup, but in a super-r-ample semigroup S, the relation $\mathscr{R}^{*,\sim}$ is a left congruence on S.

A super-r-ample semigroup S is called a completely $\mathscr{J}^{*,\sim}$-simple semigroup if S is $\mathscr{J}^{*,\sim}$-simple. Clearly, a completely $\mathscr{J}^{*}$-simple semigroup must be completely $\mathscr{J}^{*,\sim}$-simple. From the existence of left cancellative monoids which are not cancellative, we can see that completely $\mathscr{J}^{*}$-simple semigroups are the proper subclass of completely $\mathscr{J}^{*,\sim}$-simple semigroups.

In the paper, we will investigate the structure of completely $\mathscr{J}^{*,\sim}$-simple semigroups. The result which we obtained will further extend the celebrated Rees theorem for completely simple semigroups.

We first give some interesting properties of super-r-ample semigroups.

Lemma 1 Let S be a super-r-ample semigroup and let a,b be regular elements of S. Then $a\mathscr{J}^{*,\sim}b$ implies $a\mathscr{D}b$.

Proof Suppose that the elements a and b are regular in a super-r-ample semigroup S. If $a\mathscr{J}^{*,\sim}b$, then by Theorem 4.12 in Reference [4], there exists an element $c\in S$ such that $a\mathscr{L}^{*}c\mathscr{R}b$. This implies that c is regular. By the dual results of Theorem 3.7 in Reference [4], we have $a\mathscr{L}c\mathscr{R}b$. Hence $a\mathscr{D}b$.

Lemma 2 Let S be a super-r-ample semigroup. Then every regular element of S is completely regular.

Proof Let a be a regular element of S. Since S is super-r-ample, there exists an idempotent $e\in S$ such that $a\in H_e^{*,\sim}$, by Theorem 3.7 in Reference [4] and its dual, we have $a\mathscr{H}e$. This shows that a is completely regular.

Lemma 3 Let e,f be idempotents of a super-r-ample semigroup S. Then the following statements hold:

(i) If $a\mathscr{R}^{*,\sim}b$ such that $a\in H_e^{*,\sim}$ and $b\in H_f^{*,\sim}$, then for any regular element $c\in H_a^{*,\sim}$, there exists a unique inverse element c' of c in $H_b^{*,\sim}$ such that $cc'=f$ and $c'c=e$.

(ii) If $a\mathscr{L}^{*,\sim}b$ such that $a\in H_e^{*,\sim}$ and $b\in H_f{}^{*,\sim}$, then for any regular element $c\in H_a^{*,\sim}$, there exists a unique inverse element c' of c in $H_b^{*,\sim}$ such that $cc'=e$ and $c'c=f$.

Proof Suppose that $a\mathscr{R}^{*,\sim}b$ with $a\in H_e^{*,\sim}$ and $b\in H_f{}^{*,\sim}$, where e,f are idempotents of S. Then it is clear that $e\mathscr{R}^{*,\sim}f$ and hence $e\mathscr{R}f$. In particular, for any regular element $c\in H_a^{*,\sim}$, we have $c\mathscr{H}e$. Thus by Theorem 3.5 of Chapter II in Reference [1], the $\mathscr{H}$-class H_f contains a unique inverse element c' of c such that $cc'=f$ and $c'c=e$. Clearly, $H_f\subseteq H_f{}^{*,\sim}=H_b^{*,\sim}$ and hence (i) is proved.

The proof of (ii) is similar to (i).

Lemma 4 Suppose that $a\mathscr{R}^{*,\sim}b$ holds on a super-r-ample semigroup S. Then there exist some regular elements $c\in H_a^{*,\sim}$ and $c'\in H_b^{*,\sim}$ such that the right translations $\rho_c\mid L_b^{*,\sim}: x\longmapsto x\rho_c=xc$ and $\rho_{c'}\mid L_a^{*,\sim}: x\longmapsto x\rho_{c'}=xc'$ are mutually inverse $\mathscr{R}^{*,\sim}$-preserving bijections which map $L_b^{*,\sim}$ onto $L_a^{*,\sim}$ and $L_a^{*,\sim}$ onto $L_b^{*,\sim}$, respectively.

Proof Suppose that S is a super-r-ample semigroup and $a\mathscr{R}^{*,\sim}b$ in S. Then by definition, there exist idempotents e and f such that $a\in H_e^{*,\sim}$, $b\in H_f{}^{*,\sim}$. Let c be a regular element in $H_e^{*,\sim}=H_a^{*,\sim}$. Then, by Lemma 3, there exists a unique inverse element c' of c in $H_b^{*,\sim}$ such that $cc'=f$, $c'c=e$.

If $x\in L_b^{*,\sim}$, then it is clear that $x\mathscr{L}^{*,\sim}f$. But since $\mathscr{L}^{*,\sim}$ is a right congruence on S, we have

$xc\mathscr{L}^{*,\sim}fc=c$ and so $x\rho_c=xc\in L_c^{*,\sim}=L_a^{*,\sim}$. This shows that $\rho_c\mid L_b^{*,\sim}$ defined by $x\longmapsto x\rho_c=xc$ is a mapping which maps $L_b^{*,\sim}$ into $L_a^{*,\sim}$. Similarly, we can also show that $\rho_{c'}\mid L_a^{*,\sim}: x\longmapsto x\rho_{c'}=xc'$ is a mapping which maps $L_a^{*,\sim}$ into $L_b^{*,\sim}$. We now consider the composite mapping $\rho_c\mid L_b^{*,\sim}\circ\rho_{c'}\mid L_a^{*,\sim}: L_b^{*,\sim}\longrightarrow L_b^{*,\sim}$. Then, for any $x\in L_b^{*,\sim}$, we have $x\rho_c\rho_{c'}=xcc'=xf=x$, since $x\mathscr{L}^{*,\sim}b\mathscr{L}^{*,\sim}f$ and f act as a right identity element in its $\mathscr{L}^{*,\sim}$-class $L_b^{*,\sim}$. Thereby, we see that $\rho_c\mid L_b^{*,\sim}$ and $\rho_{c'}\mid L_a^{*,\sim}$ are mutually inverse bijections which map $L_b^{*,\sim}$ onto $L_a^{*,\sim}$ and $L_a^{*,\sim}$ onto $L_b^{*,\sim}$, respectively.

To show that these mappings actually preserve the $\mathscr{R}^{*,\sim}$-classes, we suppose that $x_1\mathscr{R}^{*,\sim}x_2$ for any $x_1, x_2\in L_b^{*,\sim}$. Now, we claim that $x_1c\mathscr{R}^{*,\sim}x_2c$. If $ex_1c=x_1c$ for any $e\in E(S)$, then $ex_1=ex_1cc'=x_1cc'=x_1$, since $x_1\mathscr{L}^{*,\sim}b\mathscr{L}^{*,\sim}f=cc'$. Thus, by $x_1\mathscr{R}^{*,\sim}x_2$, we immediately have $ex_2=x_2$ and hence $ex_2c=x_2c$. Similarly, if $ex_2c=x_2c$ for any $e\in E(S)$, then $ex_1c=x_1c$, and hence $x_1c\mathscr{R}^{*,\sim}x_2c$, by the definition of $\mathscr{R}^{*,\sim}$. This means the right translation of $\rho_c\mid L_b^{*,\sim}$ is $\mathscr{R}^{*,\sim}$-preserving bijection which maps $L_b^{*,\sim}$ onto $L_a^{*,\sim}$. By using similar arguments, we can also prove that $\rho_{c'}\mid L_a^{*,\sim}: x\longmapsto x\rho_{c'}=xc'$ is $\mathscr{R}^{*,\sim}$-preserving bijection which maps $L_a^{*,\sim}$ onto $L_b^{*,\sim}$. Thus, our proof is completed.

Using the similar method, we can prove the following result.

Lemma 5 Let S be a super-r-ample semigroup and $a\mathscr{L}^{*,\sim}b$ holds on S. Then there exist regular elements $c\in H_a^{*,\sim}$ and $c'\in H_b^{*,\sim}$ such that the left translations $\lambda_c\mid R_b^{*,\sim}: x\longmapsto x\lambda_c=cx$ and $\lambda_{c'}\mid R_a^{*,\sim}: x\longmapsto x\lambda_{c'}=c'x$ are mutually inverse $\mathscr{L}^{*,\sim}$-preserving bijections which map $R_b^{*,\sim}$ onto $R_a^{*,\sim}$ and $R_a^{*,\sim}$ onto $R_b^{*,\sim}$, respectively.

Definition 1 Let $\mu(T; I, \Lambda; P)$ be a Rees matrix semigroup and P the $\Lambda\times I$ matrix over a left cancellative monoid T. Then P is said to be normalized at 1 if there is an element $1\in I\cap\Lambda$ such that $P_{1i}=P_{\lambda 1}=e$, for all $i\in I$ and $\lambda\in\Lambda$, where e is the identity of the left cancellative monoid T. Furthermore, the Rees matrix semigroup $\mu(T; I, \Lambda; P)$ is called normalized if P is normalized.

Lemma 6 Let S be a completely $\mathscr{J}^{*,\sim}$-simple semigroup. Then all $\mathscr{H}^{*,\sim}$-classes of S are isomorphic left cncellative monoids.

Proof By definition and Theorem 5.1 in Reference [4], we can see that every $\mathscr{H}^{*,\sim}$-class of S is a left cancellative monoid, we only need to prove that they are isomorphic.

Let $H_a^{*,\sim}$ and $H_b^{*,\sim}$ be any two $\mathscr{H}^{*,\sim}$-classes of a semigroup S containing a and b, respectively. Then, by definition and Theorem 4.12 in Reference [4], we have $a\mathscr{D}^{*,\sim}b$, and hence there exists an element d such that $a\mathscr{R}^{*,\sim}d\mathscr{L}^{*,\sim}b$. It is clear that there exist idempotents e, g and f such that $H_a^{*,\sim}=H_e^{*,\sim}$, $H_d^{*,\sim}=H_g^{*,\sim}$, and $H_b^{*,\sim}=H_f^{*,\sim}$, since S is super-r-ample.

By Lemma 3, we can see that, for any regular element $c\in H_d^{*,\sim}$, there exists a unique inverse element c^* of c in $H_b^{*,\sim}$ such that $cc^*=g$. By Lemma 4 and Lemma 5, it follows that $\rho_c\mid H_a^{*,\sim}: x\longmapsto x\rho_c=xc$ is a bijection that maps $H_a^{*,\sim}$ onto $H_d^{*,\sim}$ and $\lambda_c^*\mid H_d^{*,\sim}: x\longmapsto x\lambda_c^*=c^*x$ is a bijection that maps $H_d^{*,\sim}$ onto $H_b^{*,\sim}$. So the mapping $\varphi=\rho_c\lambda_c^*$ is a bijection that maps $H_a^{*,\sim}$ onto $H_b^{*,\sim}$. Next, we prove the mapping φ is an isomorphism. For any $x_1, x_2\in H_a^{*,\sim}$, we have $x_1\varphi\, x_2\varphi=c^*x_1cc^*x_2c=c^*x_1gx_2c=c^*x_1x_2c=(x_1x_2)\varphi$. This proves that φ is an isomorphism.

Lemma 7 Let $S=\mu(T; I, \Lambda; P)$ be a Rees matrix semigroup over a left cancellative monoid T where each entry in P is a unit of T. Then, the following statements hold:

(i) $(i, x, \lambda)\mathscr{L}^{*,\sim}(j, y, \nu)$ if and only if $\lambda=\nu$;

(ii) $(i, x, \lambda)\mathscr{R}^{*,\sim}(j, y, \nu)$ if and only if $i=j$;

(iii) if $ab=ac$ and $ba=ca$ for some elements a, b, c in the Rees matrix semigroup $S=\mu(T; I, \Lambda; P)$, then

$b=c$.

Proof It is easy to see that $E(S)=\{(i, p_{\lambda i}^{-1}, \lambda), p_{\lambda i}^{-1} \in P\}$. If $(i, x, \lambda)\mathscr{L}^{*,\sim}(j, y, \nu)$, then by

$$(i, x, \lambda)(i, p_{\lambda i}^{-1}, \lambda)=(i, x, \lambda)$$

we have $(j, y, \nu)(i, p_{\lambda i}^{-1}, \lambda)=(j, y, \nu)$, but $(j, y, \nu)(i, p_{\lambda i}^{-1}, \lambda)=(-, -, \lambda)$, so $\lambda=\nu$.

Suppose that $\lambda=\nu$. For any $(k_1, c_1, \mu_1), (k_2, c_2, \mu_2) \in S$, $(i, x, \lambda)(k_1, c_1, \mu_1)=(i, x, \lambda)(k_2, c_2, \mu_2)$ implies that $xp_{\lambda k_1}c_1=xp_{\lambda k_2}c_2$, $\mu_1=\mu_2$. Since T is a left cancellative monoid, $p_{\lambda k_1}c_1=p_{\lambda k_2}c_2$, $\mu_1=\mu_2$, and hence $(j, y, \lambda)(k_1, c_1, \mu_1)= (j, y, \lambda)(k_2, c_2, \mu_2)$. Thus we have proved that $(i, x, \lambda)\mathscr{L}^{*,\sim}(j, y, \nu)$ if and only if $\lambda= \nu$.

If $(i, x, \lambda)\mathscr{R}^{*,\sim}(j, y, \nu)$, then by $(i, p_{\lambda i}^{-1}, \lambda)(i, x, \lambda)=(i, x, \lambda)$, we have $(i, p_{\lambda i}^{-1}, \lambda)(j, y, \nu)=(j, y, \nu)$, so $i=j$. Conversely, if $i=j$, then for any idempotent $(k, p_{\mu k}^{-1}, \mu)$ of S, $(k, p_{\mu k}^{-1}, \mu)(i, x, \lambda)=(i, x, \lambda)$ if and only if $k=i$, $k=i$ if and only if $(k, p_{\mu k}^{-1}, \mu)(j, y, \nu)=(j, y, \nu)$.

It is easy to prove that (iii) holds since T is a left cancellative monoid.

Theorem 1 Let T be a left cancellative monoid with an identity element e and I, Λ be non-empty sets. Let $P=(p_{\lambda i})$ be a $\Lambda \times I$ matrix where each entry in P is a unit of T. Suppose that P is normalized at $1 \in I \cap \Lambda$. Then the normalized Rees matrix semigroup $M=\mu(T; I, \Lambda; P)$ is a completely $\mathscr{J}^{*,\sim}$-simple semigroup.

Conversely, every completely $\mathscr{J}^{*,\sim}$-simple semigroup is isomorphic to a normalized Rees matrix semigroup $M=\mu(T; I, \Lambda; P)$ over a left cancellative monoid T.

Proof If $M=\mu(T; I, \Lambda; P)$ is a normalized Rees matrix semigroup, by Lemma 7, it is easy to prove that M is completely $\mathscr{J}^{*,\sim}$-simple.

Let M be a completely $\mathscr{J}^{*,\sim}$-simple semigroup. To begin the process of shaping M into the form of a Rees matrix semigroup, let us denote the set of $\mathscr{R}^{*,\sim}$-classes of M by I and the set of $\mathscr{L}^{*,\sim}$-classes of M by Λ, and also we write the $\mathscr{R}^{*,\sim}$-classes as $R_i^{*,\sim}(i \in I)$ and the $\mathscr{L}^{*,\sim}$-classes as $L_\lambda^{*,\sim}(\lambda \in \Lambda)$. The $\mathscr{H}^{*,\sim}$-classes $R_i^{*,\sim} \cap L_\lambda^{*,\sim}$ will be written as $H_{i\lambda}^{*,\sim}$. Then, by Lemma 6, it is known that each $\mathscr{H}^{*,\sim}$-class of M is isomorphic to a left cancellative monoid. In an arbitrary way we choose and fix an i and a λ such that $H_{i\lambda}^{*,\sim}$ is a left cancellative monoid. There is no loss of generality at all and considerable notational simplification, if we write this chosen $\mathscr{H}^{*,\sim}$-class as $H_{11}^{*,\sim}$. We shall denote its identity element by e.

Now, we claim that there exist a completely regular element $r_i \in H_{i1}^{*,\sim}$ and a completely regular element q_λ such that $q_\lambda r_1=q_1 r_i=e$ for each $i \in I$ and each $\lambda \in \Lambda$. In fact, for any completely regular element $c \in H_{11}^{*,\sim}$, we can let $r_1=c$ and $q_1=c'$ which is the unique inverse element of c in $H_{11}^{*,\sim}$. Hence, for each $\lambda \in \Lambda$, by Lemma 3 (i), $H_{1\lambda}^{*,\sim}$ contains a unique inverse element q_λ of r_1 such that $q_\lambda r_1=e$. Similarly, for each $i \in I$, we can choose a completely regular element $r_i \in H_{i1}^{*,\sim}$ such that $q_1 r_i=e$. Thus, we have chosen the sets $\{q_\lambda \mid \lambda \in \Lambda\}$ and $\{r_i \mid i \in I\}$, where q_λ, r_i are both completely regular elements of S. Now, by Lemma 4, we know that for any $x \in H_{11}^{*,\sim}$, the mapping $x \longmapsto xq_\lambda$ is a bijection from $H_{11}^{*,\sim}$ to $H_{1\lambda}^{*,\sim}$. Similarly, by Lemma 5, the mapping $y \longmapsto r_i y$ $(y \in H_{1\lambda}^{*,\sim})$ is a bijection from $H_{1\lambda}^{*,\sim}$ to $H_{i\lambda}^{*,\sim}$. Thus, we obtain a unique expression $r_i x q_\lambda (x \in H_{11}^{*,\sim})$ for each element of $H_{i\lambda}^{*,\sim}$.

Now, we let $H_{11}^{*,\sim}=T$ and define $P=(p_{\lambda i})$ to be the $\Lambda \times I$ matrix with $p_{\lambda i}=q_\lambda r_i$ for any $(\lambda, i) \in \Lambda \times I$. Suppose that f is the identity element of $H_{i\lambda}^{*,\sim}$. Then, $r_i \mathscr{R}^{*,\sim} f$ and so $fr_i=r_i$ since f is a left identity in its $\mathscr{R}^{*,\sim}$-class. By Lemma 4 again, the mapping $x \longmapsto xr_i$ is an $\mathscr{R}^{*,\sim}$-preserving bijection from L_λ^* onto L_1^*, and thereby, $p_{\lambda i}=q_\lambda r_i \in H_{11}^{*,\sim}$. Notice that q_λ and r_i are completely regular elements of S. Then, by the definition of M and Lemma 2, $p_{\lambda i}$ is a completely regular element and so it is a unit of T. Thus we obtain a normalized Rees matrix semigroup $\mu(T; I, \Lambda; P)$ over a left cancellative monoid T, that is, each entry in P is a unit of T and P is also normalized at $1 \in I \cap \Lambda$.

We now consider the mapping $\varphi: I \times T \times \Lambda \longrightarrow S$ defined by $(i, a, \lambda)\varphi = r_i a q_\lambda$, for any $a \in T$. Clearly, φ is a bijection because $S = \bigcup \{H^{*,\sim}_{i\lambda} \mid (i, \lambda) \in I \times \Lambda\}$ and by routine checking, φ is an isomorphism which maps the normalized Rees matrix semigroup $\mu(T; I, \Lambda; P)$ onto the completely $\mathscr{J}^{*,\sim}$-simple semigroup S.

Remark 1 If S is an abundant semigroup, then $\mathscr{R}^{*,\sim} = \mathscr{R}^*$. Hence S is a completely $\mathscr{J}^{*,\sim}$-simple semigroup if and only if S is a completely $\mathscr{J}^*$-simple semigroup; S is a left cancellative monoid if and only if S is a cancellative monoid.

If S is a regular semigroup, then $\mathscr{R}^{*,\sim} = \mathscr{R}$, $\mathscr{L}^{*,\sim} = \mathscr{L}$. Hence S is a completely $\mathscr{J}^{*,\sim}$-simple semigroup if and only if S is a completely simple semigroup; S is a left cancellative monoid if and only if S is a group.

Thus, Rees theorem in Reference [1] and Theorem 3.4 in Reference [3] can be regard as direct consequences of our Theorem 1.

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完全 $\mathscr{J}^{*,\sim}$-单半群的结构

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摘要： 完全 $\mathscr{J}^{*,\sim}$-单半群是完全单半群在 rpp 半群中的推广．借助左可消幺半群上的正规 Rees 矩阵半群，建立了完全 $\mathscr{J}^{*,\sim}$-单半群的结构．

关键词： r-ample 半群；super-r-ample 半群；完全 $\mathscr{J}^{*,\sim}$-单半群；正规 Rees 矩阵半群

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