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Existence of the Periodic Solutions for the Complex Gross-Pitaevskii Equation Modeled in the Collapse of BECs①

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Abstract: The complex Gross-Pitaevskii equation (CGPE) modeled in the collapse of BECs with attractive interactions is studied. The existence and uniqueness of the periodic solution is obtained for the CGPE by using the Leray-Schauder fixed point theorem and the Galerkin method.

Key words: complex Gross-Pitaevskii equation; collapse; approximate solution; Galerkin method

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Consider the following complex Gross-Pitaevskii equation (CGPE) modeled in the collapse of attractive BECs

$$\begin{cases} \mathrm{i}u_t = -\Delta u + |x|^2 u + \alpha |u|^2 u - \mathrm{i}\beta u - \mathrm{i}\gamma |u|^4 u & (t, x) \in \mathbf{R}_+ \times \Omega \\ u(0, x) = u_0(x) & x \in \Omega \\ u(t, e_j \cdot x + L) = u(t, e_j \cdot x) \quad j = 1, 2, 3 & (t, x) \in \mathbf{R}_+ \times \Omega \end{cases} \tag{1}$$

where $\Omega = (0, L) \times (0, L) \times (0, L) \in \mathbf{R}^3$, $u: \mathbf{R}_+ \times \Omega \longrightarrow \mathbf{C}$ is a complex value wave function, $\alpha, \beta, \gamma > 0$ and e_j is the j coordinate basis of $\mathbf{R}^3$. Let $H^k_{\mathrm{per}}(\Omega) = \{u \in H^k(\Omega): u \text{ is a periodic function}\}$.

Recently, many experiments on this phenomenon have been done and one of the interesting problems is the behavior of BECs. References [1-2] show that when the number of atoms exceeds the critical value, the collapse of the condensate takes places with the phenomenological three-body loss term which is modeled by equation (1). As we know, there is few results for equation (1) with the harmonic potential involving the complex power type nonlinearity $-\mathrm{i}\beta u - \mathrm{i}\gamma |u|^4 u$, in which the last one is H^1 critical different from the method in references [3-4], but this is our subject in this paper.

Let $u_0 \in H^2_{\mathrm{per}}(\Omega)$ and $\{\phi_j\}_{j=1}^{\infty}$ be the normalized orthonormal basis of $H^2_{\mathrm{per}}(\Omega)$. By the Galerkin method, the approximate solutions of equation (1) are denoted by $u_n(t) = \sum_{k=1}^{n} d_{kn}(t) \phi_k(x)$, and $u_n(t)$ satisfies the following equation:

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$$\begin{cases}(\mathrm{i}u_{nt},\ \phi_j)=(-\Delta u_n+|x|^2u_n+\alpha|u_n|^2u_n,\ \phi_j)-\mathrm{i}(\beta u_n+\gamma|u_n|^4u_n,\ \phi_j) & j=1,2,\cdots,n\\ u_n(t,\ e_j\cdot x+L)=u_n(t,\ e_j\cdot x) & (t,\ x)\in\mathbf{R}_+\times\Omega\\ u_n(0,\ x)=P_nu_0(x) & x\in\Omega\end{cases}\tag{2}$$

where P_n is the orthogonal projection of $H^2_{\mathrm{per}}(\Omega)$ on $H_n=\mathrm{span}\{\phi_1,\ \phi_2,\ \cdots,\ \phi_n\}$. Define the mapping $F_\lambda:\ v_n\longmapsto u_n$ as

$$(\mathrm{i}u_{nt},\ \phi_j)=(-\Delta u_n+|x|^2u_n+\alpha|u_n|^2u_n,\ \phi_j)-\mathrm{i}(\lambda(\beta v_n+\gamma|v_n|^4v_n),\ \phi_j)\tag{3}$$

where $j=1,2,\cdots,n$ and $\lambda\in[0,\ 1]$. It is easy to prove that $F_\lambda\in C^1(H_n)$ is the continuous embedding mapping. If $\lambda=0$, clearly, the approximate solution $u_n(t)$ exists. Now, in order to get the solutions of equation (3), we only need to prove that $\|u_n(t)\|$ is bounded. Throughout this paper, C,C_j,K_j denote various positive constants. If necessary, $C_j(\cdot,\cdot)$ and $K_j(\cdot,\cdot)$ denote the dependence relationship.

Lemma 1 Suppose that $u_0\in H^2_{\mathrm{per}}(\Omega)$, $F_\lambda(u_n)=u_n$ and $\lambda\in[0,\ 1]$. Then equation (2) possesses a solution. Furthermore, there exist $K_1(u_0),K_2(\beta,\ u_0),K_3(\gamma,\ u_0)>0$ independent of n and λ such that, for any $T>0$, one has

$$\|u_n(T,\ x)\|_2^2\leqslant K_1(u_0)\qquad\int_0^T\|u_n(t,\ x)\|_2^2\mathrm{d}t\leqslant K_2(\beta,\ u_0)$$

$$\int_0^T\|u_n(t,\ x)\|_6^6\mathrm{d}t\leqslant K_3(\gamma,\ u_0)$$

Proof By $F_\lambda(u_n)=u_n$, we have

$$(\mathrm{i}u_{nt},\ u_n)=(-\Delta u_n+|x|^2u_n+\alpha|u_n|^2u_n,\ u_n)-\mathrm{i}(\lambda(\beta u_n+\gamma|u_n|^4u_n),\ u_n)$$

Taking its imaginary part, we get

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_n\|_2^2+\lambda\gamma\|u_n\|_6^6+\lambda\beta\|u_n\|^2=0\tag{4}$$

This yields $\frac{\mathrm{d}}{\mathrm{d}t}\|u_n\|_2^2\leqslant0$. So $\|u_n(t)\|^2$ is decreasing with respect to t and

$$\|u_n(T)\|_2^2\leqslant\|u_n(0,\ x)\|_2^2=\|P_nu_0(x)\|_2^2=K_1(u_0)\qquad T>0.$$

By the Leray-Schauder fixed point theorem, we have $F_\lambda(u_n)=u_n$ and $\lambda=1$. Thus, equation (2) possesses a solution.

For any $T>0$, integrating from 0 to T on equation (4) with $\lambda=1$, we have

$$2\gamma\int_0^T\|u_n(t)\|_6^6\mathrm{d}t+2\beta\int_0^T\|u_n(t)\|_2^2\mathrm{d}t=\|u_n(0,\ x)\|_2^2-\|u_n(T,\ x)\|_2^2\leqslant\|u_n(0,\ x)\|_2^2$$

which yields Lemma 1.

Lemma 2 Suppose that $u_0\in H^2_{\mathrm{per}}(\Omega)$. For any $T>0$, there exist $K_4(\gamma,\ \alpha,\ T,\ u_0)>0$ and $K_5(\gamma,\ \alpha,\ \beta,\ T,\ u_0)>0$ such that

$$\|u_{nt}(T,\ x)\|_2^2\leqslant K_4(\gamma,\ \alpha,\ T,\ u_0)\qquad\int_0^T\int_\Omega|u_n|^4|u_{nt}|^2\mathrm{d}x\mathrm{d}t\leqslant K_5(\gamma,\ \alpha,\ T,\ u_0)$$

Proof By equation (2), we have

$$(\mathrm{i}u_{ntt},\ \phi_j)=(-\Delta u_{nt}+|x|^2u_{nt}+2\alpha|u_n|^2u_{nt}+\alpha u_n^2u_{nt}^*,\ \phi_j)-$$
$$\mathrm{i}(\beta u_{nt}+\gamma(3|u_n|^4u_{nt}+2|u_n|^2u_n^2u_{nt}^*),\ \phi_j)$$

Therefore, we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2=-\beta\int_\Omega|u_{nt}|^2\mathrm{d}x+\alpha\mathrm{Im}\int_\Omega u_n^2(u_{nt}^*)^2\mathrm{d}x-\gamma\mathrm{Re}\int_\Omega(3|u_n|^4|u_{nt}|^2+2|u_n|^2u_n^2(u_{nt}^*)^2)\mathrm{d}x$$

together with the Young inequality, we deduce

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2+\gamma\int_\Omega|u_n|^4|u_{nt}|^2\mathrm{d}x\leqslant$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2+\beta\int_\Omega|u_{nt}|^2\mathrm{d}x+\gamma\mathrm{Re}\int_\Omega(2|u_n|^4|u_{nt}|^2+$$

$$
\begin{aligned}
&2|u_n|^2 u_n^2 (u_{nt}^*)^2)\mathrm{d}x+\gamma\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x=\\
&\alpha\mathrm{Im}\int_\Omega u_n^2(u_{nt}^*)^2\mathrm{d}x\leqslant\\
&\alpha\int_\Omega |u_n{}^2||u_{nt}||u_{nt}|\mathrm{d}x\leqslant\\
&\gamma\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x+C(\gamma,\alpha)\|u_{nt}\|_2^2
\end{aligned}
\tag{5}
$$

which yields $\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2\leqslant C(\gamma,\alpha)\|u_{nt}\|_2^2$. Therefore, we obtain that

$$\|u_{nt}(T,x)\|_2^2\leqslant C\mathrm{e}^{C(\gamma,\alpha)T}\|u_{nt}(0,x)\|_2^2=K_4(\gamma,\alpha,T,u_0)\qquad T>0$$

where

$$
\begin{aligned}
u_{nt}(0,x)=&\mathrm{i}(\Delta P_n u_0(x)-|x|^2P_nu_0(x)-\alpha|P_nu_0(x)|^2P_nu_0(x)+\\
&\mathrm{i}\beta P_nu_0(x)+\mathrm{i}\gamma|P_nu_0(x)|^4P_nu_0(x))
\end{aligned}
$$

Furthermore, by inequality (5), we get

$$
\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2+\gamma\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x\leqslant\\
&\alpha\int_\Omega |u_n{}^2||u_{nt}||u_{nt}|\mathrm{d}x\leqslant\frac{\gamma}{2}\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x+C_1(\gamma,\alpha)\|u_{nt}\|_2^2
\end{aligned}
$$

which yields

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u_{nt}\|_2^2+\frac{\gamma}{2}\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x\leqslant C_1(\gamma,\alpha)\|u_{nt}\|_2^2 \tag{6}$$

For any $T>0$, integrating from 0 to T on inequity (6), we have

$$
\begin{aligned}
\frac{\gamma}{2}\int_0^T\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x\mathrm{d}t\leqslant&\frac{1}{2}\|u_{nt}(T,x)\|_2^2+\frac{\gamma}{2}\int_0^T\int_\Omega |u_n|^4|u_{nt}|^2\mathrm{d}x\mathrm{d}t\leqslant\\
&C_1(\gamma,\alpha)\int_0^T\|u_{nt}\|_2^2\mathrm{d}t+\frac{1}{2}\|u_{nt}(0,x)\|_2^2\leqslant\\
&C_1(\gamma,\alpha)K_4(\gamma,\alpha,T,u_0)T+\frac{1}{2}\|u_{nt}(0,x)\|_2^2
\end{aligned}
$$

which yields Lemma 2.

Lemma 3　Suppose that $u_0\in H_{\text{per}}^2(\Omega)$. Then for any $T>0$, there exist $K_6(\gamma,\alpha,\beta,T,u_0)>0$ and $K_7(\gamma,\alpha,\beta,T,u_0)>0$ such that

$$\|\nabla u_n(T,x)\|_2^2+\int_\Omega |x|^2|u_n(T,x)|^2\mathrm{d}x\leqslant K_6(\gamma,\alpha,\beta,T,u_0)$$

$$\|u_n(T,x)\|_4^4\leqslant K_7(\gamma,\alpha,\beta,T,u_0)$$

Proof　By $(\mathrm{i}u_{nt},u_{nt})=(-\Delta u_n+|x|^2u_n+\alpha|u_n|^2u_n,u_{nt})-(\mathrm{i}\beta u_n+\mathrm{i}\gamma|u_n|^4u_n,u_{nt})$, take its the real part, together with the Young inequality, we deduce

$$
\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega(|\nabla u_n|^2+|x|^2|u_n|^2+\frac{\alpha}{2}|u_n|^4)\mathrm{d}x=\mathrm{Im}\int_\Omega(\beta u_n^*u_{nt}+\gamma|u_n|^4u_n^*u_{nt})\mathrm{d}x\leqslant\\
&\frac{\beta}{2}\|u_{nt}\|_2^2+\frac{\beta}{2}\|u_n\|_2^2+\frac{\gamma}{2}\|u_n\|_6^6+\frac{\gamma}{2}\int_\Omega|u_n|^4|u_{nt}|^2\mathrm{d}x
\end{aligned}
\tag{7}
$$

where u^* is the conjugation of u. For any $T>0$, integrating inequality (7) between 0 and T, together with Lemma 1 and Lemma 2, we obtain

$$
\begin{aligned}
&\frac{1}{2}\left(\|\nabla u_n(T,x)\|_2^2+\int_\Omega|x|^2|u_n(T,x)|^2\mathrm{d}x+\frac{\alpha}{2}\|u_n(T,x)\|_4^4\right)\leqslant\\
&\frac{\beta}{2}\int_0^T\|u_{nt}\|_2^2\mathrm{d}t+\frac{\beta}{2}\int_0^T\|u_n\|_2^2\mathrm{d}t+\frac{\gamma}{2}\int_0^T\|u_n\|_6^6\mathrm{d}t+\frac{\gamma}{2}\int_0^T\int_{\mathbf{R}^3}|u_n|^4|u_{nt}|^2\mathrm{d}x\mathrm{d}t-\\
&\frac{1}{2}\left(\|\nabla u_n(0,x)\|_2^2+\int_\Omega|x|^2|u_n(0,x)|^2\mathrm{d}x+\frac{\alpha}{2}\|u_n(0,x)\|_4^4\right)\leqslant
\end{aligned}
$$

$$\frac{\beta}{2}K_4(\gamma,\alpha,T,u_0)T+\frac{\beta}{2}K_2(\beta,u_0)+\frac{\gamma}{2}K_3(\gamma,u_0)+\frac{\gamma}{2}K_5(\gamma,\alpha,T,u_0)-$$
$$\frac{1}{2}\left(\|\nabla P_n u_0(x)\|_2^2+\int_{\mathbf{R}^3}|x|^2|P_n u_0(x)|^2\mathrm{d}x+\frac{\alpha}{2}\|P_n u_0(x)\|_4^4\right)$$

which yields Lemma 3.

Lemma 4 Suppose that $u_0\in H^2_{\mathrm{per}}(\Omega)$. Then for any $T>0$, there exists $K_8(\gamma,\alpha,\beta,T,u_0)>0$ such that $\|\Delta u_n\|_2^2\leqslant K_8(\gamma,\alpha,\beta,T,u_0)$.

Proof By $(\Delta u_n,\Delta u_n)=(-\mathrm{i}u_{nt}+|x|^2u_n+\alpha|u_n|^2u_n,\Delta u_n)-\mathrm{i}(\beta u_n+\gamma|u_n|^4u_n,\Delta u_n)$, we obtain

$$\begin{aligned}\|\Delta u_n\|_2^2=&-\mathrm{i}\int_\Omega u_{nt}\Delta u_n^*\mathrm{d}x+\int_\Omega|x|^2u_n\Delta u_n^*\mathrm{d}x+\alpha\int_\Omega|u_n|^2u_n\Delta u_n^*\mathrm{d}x-\mathrm{i}\beta\int_\Omega u_n\Delta u_n^*\mathrm{d}x\\&-\mathrm{i}\gamma\int_\Omega|u_n|^4u_n\Delta u_n^*\mathrm{d}x=\\&-\mathrm{i}\int_\Omega u_{nt}\Delta u_n^*\mathrm{d}x-\int_\Omega|x|^2|\nabla u_n|^2\mathrm{d}x-2\int_\Omega u_n x\cdot\nabla u_n^*\mathrm{d}x+\mathrm{i}\beta\int_\Omega|\nabla u_n|^2\mathrm{d}x\\&-\alpha\int_\Omega(2|u_n|^2|\nabla u_n|^2+u_n^2(\nabla u_n^*)^2)\mathrm{d}x+\mathrm{i}\gamma\int_\Omega(3|u_n|^4|\nabla u_n|^2+2u_n^3u_n^*(\nabla u_n^*)^2)\mathrm{d}x\end{aligned}\tag{8}$$

Take the imaginary part of equation (8), together with the Hölder inequality, we get

$$\begin{aligned}0\geqslant&-\int_\Omega|u_{nt}||\Delta u_n|\mathrm{d}x-2\mathrm{Im}\int_\Omega u_n x\cdot\nabla u_n^*\mathrm{d}x-\alpha\mathrm{Im}\int_\Omega u_n^2(\nabla u_n^*)^2\mathrm{d}x+\\&\gamma\mathrm{Re}\int_\Omega(2|u_n|^4|\nabla u_n|^2+2u_n^3u_n^*(\nabla u_n^*)^2)\mathrm{d}x+\gamma\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\geqslant\\&-\|u_{nt}\|_2\|\Delta u_n\|_2-2\int_\Omega|xu_n||\nabla u_n|\mathrm{d}x-\alpha\int_\Omega|u_n|^2|\nabla u_n||\nabla u_n|\mathrm{d}x+\gamma\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\geqslant\\&-\|u_{nt}\|_2\|\Delta u_n\|_2-\int_\Omega|x|^2|u_n|^2\mathrm{d}x-\|\nabla u_n\|_2^2-\frac{\gamma}{2}\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x-\\&C_2(\gamma,\alpha)\|\nabla u_n\|_2^2+\gamma\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x=\\&-\|u_{nt}\|_2\|\Delta u_n\|_2-\int_\Omega|x|^2|u_n|^2\mathrm{d}x-(1+C_2(\gamma,\alpha))\|\nabla u_n\|_2^2+\frac{\gamma}{2}\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\end{aligned}\tag{9}$$

For any $\varepsilon>0$, from inequality (9), Lemma 2 and Lemma 3, and the Young inequality, we deduce

$$\begin{aligned}\frac{\gamma}{2}\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\leqslant&\|u_{nt}\|_2\|\Delta u_n\|_2+\int_\Omega|x|^2|u_n|^2\mathrm{d}x+(1+C_2(\gamma,\alpha))\|\nabla u_n\|_2^2\mathrm{d}x\leqslant\\&\frac{2}{\gamma\varepsilon}\|u_{nt}\|_2^2+\frac{\gamma\varepsilon}{2}\|\triangle u_n\|_2^2+K_6(\gamma,\alpha,T,\beta,u_0)(2+C_2(\gamma,\alpha))\leqslant\\&\frac{2}{\gamma\varepsilon}K_4(\gamma,\alpha,T,u_0)+\frac{\gamma\varepsilon}{2}\|\triangle u_n\|_2^2+K_6(\gamma,\alpha,T,\beta,u_0)(2+C_2(\gamma,\alpha))\end{aligned}\tag{10}$$

It follows from inequality (10) that there exists $K_{10}(\varepsilon,\gamma,\alpha,\beta,T,u_0)>0$ such that

$$\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\leqslant K_9(\varepsilon,\gamma,\alpha,\beta,T,u_0)+\varepsilon\|\triangle u_n\|_2^2$$

Take the real part of equation (8), together with Lemma 1, Lemma 2 and Lemma 3, we deduce

$$\begin{aligned}\|\Delta u_n\|_2^2\leqslant&\|u_{nt}\|_2^2+6\|u_n\|_2^2+4\gamma\int_\Omega|u_n|^4|\nabla u_n|^2\mathrm{d}x\leqslant\\&K_4(\gamma,\alpha,T,u_0)+6K_1(u_0)+4\gamma K_9(\varepsilon,\gamma,\alpha,\beta,T,u_0)+4\gamma\varepsilon\|\Delta u_n\|_2^2\end{aligned}$$

Choosing $\varepsilon=\frac{1}{8\gamma}$, this yields Lemma 4.

Theorem 1 Suppose that $u_0\in H^2_{\mathrm{per}}(\Omega)$. Then there exists a unique periodic solution u of equation (1) satisfying $u\in C^1([0,\infty),L^2(\Omega))\cap C([0,\infty),H^2_{\mathrm{per}}(\Omega))$.

Proof　From Lemma 4 and the imbedding theorem, we obtain that $\| u_n \|_{L^\infty}$ is bounded. Together with Lemma 1, Lemma 2 and Lemma 3, we obtain the uniformly prior estimation of the approximate solutions of equation (1). And we know that there exists a subsequence $\{u_{n_k}(t, x)\}$ satisfying that $u_{n_k}(t, \cdot) \rightharpoonup u$ in $L^\infty((0, T], H^2_{\mathrm{per}}(\Omega))$ and u is the solution of equation (1).

Suppose that u_1 and u_2 are two solutions of equation (1), then we obtain

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|(u_1-u_2)\|_2^2 = {} & \alpha \mathrm{Im}\int_\Omega (|u_1|^2 u_1 - |u_2|^2 u_2)(u_1-u_2)^* \mathrm{d}x - \beta\int_\Omega |u_1-u_2|^2 \mathrm{d}x - \\ & \gamma \mathrm{Re}\int_\Omega (|u_1|^4 u_1 - |u_2|^4 u_2)(u_1-u_2)^* \mathrm{d}x \leqslant \\ & 2\alpha\int_\Omega \sup\{|u_1|^2, |u_2|^2\}|u_1-u_2|^2 \mathrm{d}x + 2\gamma\int_\Omega \sup\{|u_1|^4, |u_2|^4\}|u_1-u_2|^2\mathrm{d}x \leqslant \\ & C\sup\{|u_1|^2_{L^\infty}, |u_2|^2_{L^\infty}, |u_1|^4_{L^\infty}, |u_2|^4_{L^\infty}\}\|u_1-u_2\|_2^2 \leqslant \\ & C\sup\{\|u_1\|_{H^2}, \|u_2\|_{H^2}\}\|u_1-u_2\|_2^2\end{aligned}$$

Thus, we have

$$\begin{aligned}0 \leqslant {} & \|u_1(t, x) - u_2(t, x)\|_2^2 \leqslant \\ & C\mathrm{e}^{\tilde{k}T}\|u_1(0, x) - u_2(0, x)\|_2^2 = C\mathrm{e}^{\tilde{k}T}\|u_0(x) - u_0(x)\|_2^2 = 0\end{aligned}$$

for any $T>0$, $t\in[0, T]$, where $\tilde{k} = 2C\sup\{\|u_1\|_{H^2}, \|u_2\|_{H^2}\}$. Thus $u_1(t, x) = u_2(t, x)$.

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BECs 中坍塌现象的复 Gross-Pitaevskii 模型方程周期解的存在性

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摘要：用 Leray-Schauder 不动点定理及 Galerkin 方法对具有吸引作用的 BECs 中坍塌现象的复 Gross-Pitaevskii 模型方程(CGPE)进行研究，得到了 CGPE 周期解的存在唯一性结果.

关键词：复 Gross-Pitaevskii 方程；坍塌；近似解；Galerkin 方法

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